Gestures, Coupling Graphs, and Strings WorMTRiP 2022

Scott Nelson

Stony Brook University

September 23, 2022





Phonological Representations

- Representations are central to modern phonological theory.
- Many different proposals across phonological domains.



Dresher (2009); Archangeli (1988); Clements (1985); Goldsmith (1976); Inkelas and Shih (2016); Browman and Goldstein (1986); among many others

Segmental Representations



- Many different proposals within the generative tradition
- Focus of this talk will be on segmental representations

- Previous research has used model theory to compare different proposed representations.
 - Syllable Representations
 - Tonal Geometry
 - Autosegmental/Q Theory
 - Feature Systems

Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

Previous research has used model theory to compare different proposed representations.

- Syllable Representations
- Tonal Geometry
- Autosegmental/Q Theory
- Feature Systems



Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

- Previous research has used model theory to compare different proposed representations.
 - Syllable Representations
 - Tonal Geometry
 - Autosegmental/Q Theory
 - Feature Systems



Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

- Previous research has used model theory to compare different proposed representations.
 - Syllable Representations
 - Tonal Geometry
 - Autosegmental/Q Theory
 - Feature Systems





Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

- Previous research has used model theory to compare different proposed representations.
 - Syllable Representations
 - Tonal Geometry
 - Autosegmental/Q Theory
 - Feature Systems

| $CPL(\mathcal{M}^v)$ | $\mathcal{M}^{\upsilon}_{\mathrm{P}}$ | $\mathcal{M}_{\mathrm{F}}^{\upsilon}$ | \mathcal{M}^{v}_{c} |
|----------------------|---------------------------------------|---------------------------------------|-----------------------|
| voi | {N,D} | {N,D} | {D} |
| son | {N} | {N} | {N} |
| son∧voi | {N} | {N} | {} |
| MISSING | - | $\{D\}, \{T\}, \{D,T\}$ | ${T}, {D,T}$ |
| Extra | - | - | - |

| $CNPL(\mathcal{M}^v)$ | $\mathcal{M}_{P}^{\upsilon}$ | $\mathcal{M}_{\mathrm{F}}^{\upsilon}$ | $\mathcal{M}^v_{\mathrm{c}}$ |
|-----------------------|------------------------------|---------------------------------------|------------------------------|
| voi | {N,D} | {N,D} | {D} |
| ¬voi | {T} | {T} | $\{N,T\}$ |
| son | {N} | {N} | {N} |
| ⊐son | {D,T} | $\{D,T\}$ | $\{D,T\}$ |
| son∧¬son | {} | {} | {} |
| son∧voi | {N} | {N} | {} |
| son∧¬voi | {} | {} | {N} |
| ¬son∧voi | {D} | {D} | {D} |
| ¬son∧¬voi | {T} | {T} | {T} |
| voi∧¬voi | {} | {} | {} |
| MISSING | - | - | - |
| Extra | $\{D\}, \{T\}, \{D,T\}$ | - | $\{N,T\}$ |

Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

Cross-Theory Comparison

- All of the previous work has looked at representations within the larger tradition of generative phonology
 - roughly: input/output mappings described by symbolic changes

Browman and Goldstein (1986, 1992, 1995); Nam (2007); Goldstein (2011); Friedman and Visser (2014); Strother-Garcia (2019); Oakden (2020)

Cross-Theory Comparison

- All of the previous work has looked at representations within the larger tradition of generative phonology
 - roughly: input/output mappings described by symbolic changes
- Articulatory Phonology is a theory of phonological representations based in nonlinear dynamics.

crucially: no input/output mappings

Browman and Goldstein (1986, 1992, 1995); Nam (2007); Goldstein (2011); Friedman and Visser (2014); Strother-Garcia (2019); Oakden (2020)

Cross-Theory Comparison

- All of the previous work has looked at representations within the larger tradition of generative phonology
 - roughly: input/output mappings described by symbolic changes
- Articulatory Phonology is a theory of phonological representations based in nonlinear dynamics.
 - crucially: no input/output mappings
- In this talk I will show the *bi-interpretability* of strings and coupling graphs which are the lexical representations used in Articulatory Phonology.

Browman and Goldstein (1986, 1992, 1995); Nam (2007); Goldstein (2011); Friedman and Visser (2014); Strother-Garcia (2019); Oakden (2020)

Background

- A gesture is "a characteristic pattern of movement of an articulator (or of an articulatory subsystem) through space, over time" (p. 237)
- "We take, then, as a first hypothesis that gestures can be characterised in terms of a dynamical system and its associated motion variables and parameter values [e.g. constriction location/degree]..." (p. 240).

 $\mathbf{m}\ddot{\mathbf{x}} + \mathbf{k}(\mathbf{x} - \mathbf{x}_0) = 0$

- "...a constellation of gestures is a set of gestures that are coordinated with one another by means of phasing..." (p. 185).
- "Each gesture is assumed to be active for a fixed proportion of its virtual cycle...The linguistic gestural model uses this proportion, along with the stiffness of each gesture and the phase relations among the gestures, to calculate a *gestural score* that specifies the temporal activation intervals for each gesture in an utterance" (p. 187).
- "The parameter value specifications and activation intervals from the gestural score are input to the task-dynamical model..., which calculates the time-varying response of the tract variable and component articulators to the imposition of the dynamical regimes defined by the gestural score" (p. 188).



Representations of [pan]

- Constellation
- Gestural Score
- Trajectories

Browman and Goldstein (1986, 1995)



Representations of [pan]

- Constellation
- Gestural Score
- Trajectories

Browman and Goldstein (1986, 1995)



Representations of [pan]

- Constellation
- Gestural Score
- Trajectories

Browman and Goldstein (1986, 1995)

Coupling Graphs as Lexical Representations

"In previous work, the gestural scores were constructed using rules that specified the relative phase of pairs of gestures...In this model, planning oscillators associated with the set of gestures in a given utterance are coupled in a pairwise, bidirectional manner specified in a coupling graph (or structure) that is part of the lexical specification of a word" (p. 38).

Coupling Graphs as Lexical Representations



Articulatory Phonology vs. Gestural Representations in a Generative Phonology

- Many researchers have used gestural representations within generative phonology.
- Here, I am focused on Articulatory Phonology as a theory of phonology that does not have a generative element.
- This offers a more interesting comparison case and shows the strength of model theoretic representations as tools for theory comparison.

McMahon et al. (1994); Zsiga (1997); Gafos (2002); Hall (2003); Davidson (2004); Bateman (2007); *inter alia*

Model Theoretic Representations

Chadwick (2021) Model

Figure 2: Model Signature for Liquid Asymmetry

- Focus was specifically on liquid asymmetry.
- Only used in-phase and anti-phase binary relations
- Included extra information (alpha, mora)

Coupling Graph Models

| Relation | Label |
|-----------------------|------------|
| \diamond | In-phase |
| \triangleleft_{180} | Anti-phase |
| \triangleleft_{60} | Abutting |
| \triangleleft_{30} | Eccentric |

 4 binary relations based on common phase relations in Articulatory Phonology

Coupling Graph Models

| Relation | Label | Relation | Label |
|----------|-------------------------------|----------|-------------------------------------|
| LIPS | Labial Articulator | rel | Constriction Degree: release |
| TT | Tongue Tip Articulator | pro | Constriction Location: protruded |
| TB | Tongue Body Articulator | dent | Constriction Location: dental |
| VEL | Velum Articulator | alv | Constriction Location: alveolar |
| GLO | Glottis Articulator | palv | Constriction Location: postalveolar |
| clo | Constriction Degree: closed | pal | Constriction Location: palatal |
| crit | Constriction Degree: critical | vel | Constriction Location: velar |
| nar | Constriction Degree: narrow | uvul | Constriction Location: uvular |
| v | Constriction Degree: vowel | phar | Constriction Location: pharyngeal |
| wide | Constriction Degree: wide | - | |

► Unary labeling relations.

Coupling Graph Model: [læft]

$$\mathcal{D} := \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$\Leftrightarrow := \{(1, 2), (2, 4), (5, 9)\}$$

$$\lhd_{180} := \{(4, 5)\}$$

$$\lhd_{60} := \{(2, 3), (5, 6), (7, 8)\}$$

$$\lhd_{30} := \{(5, 7)\}$$

LIPS := $\{5, 6\}$
TT := $\{2, 3, 7, 8\}$
TB := $\{1, 4\}$
GLO := $\{9\}$

$$\begin{array}{l} \texttt{dent} := \{5\} \\ \texttt{alv} := \{2,7\} \\ \texttt{uvul} := \{1\} \\ \texttt{phar} := \{4\} \\ \texttt{clo} := \{7\} \\ \texttt{crit} := \{5\} \\ \texttt{nar} := \{1,2\} \\ \texttt{wide} := \{9\} \\ \texttt{rel} := \{3,6,8\} \\ \texttt{V} := \{4\} \end{array}$$

Coupling Graph Model: [læft]



String Model: [læft]

| Relation | Label | |
|-------------------------------------|-----------|--|
| \triangleleft | Successor | |
| $\sigma(\forall \sigma \in \Sigma)$ | Segment | |

$$\begin{aligned} \langle \mathcal{D} &:= \{1, 2, 3, 4\} \\ \lhd &:= \{(1, 2), (2, 3), (3, 4)\} \\ \mathbf{a} &:= \{2\} \\ \mathbf{f} &:= \{3\} \\ \mathbf{l} &:= \{1\} \\ \mathbf{t} &:= \{4\} \\ \sigma &:= \{\}; \sigma \in \Sigma \setminus \{\mathbf{a}, \mathbf{f}, \mathbf{l}, \mathbf{t}\} \rangle \end{aligned}$$

Translations

- We can translate between structures using monadic second order logic.
- Formulae such as $\varphi_P(x) = Q(x)$ are interpreted as "domain element *x* has property *P* in the output structure if it has property *Q* in the input structure".
- Here we'll define two translations:
 - Coupling graph to string: Γ^{sg}
 - String to coupling graph: Γ^{gs}

Courcelle (1994); Engelfriet and Hoogeboom (2001)

Identifying the "spine"



• We can identify the *spine* of a coupling graph by looking at the subgraph that does not include:

- Secondary articulations
- Release Gestures
- Glottal Gestures
- Velum Gestures



 \triangleright $C := \{1\}$



 $\begin{array}{l} & \varphi_{l}(\boldsymbol{x}) := \mathrm{TT}(\boldsymbol{x}) \wedge \mathrm{alv}(\boldsymbol{x}) \wedge \mathrm{nar}(\boldsymbol{x}) \wedge \exists \boldsymbol{y} [\boldsymbol{x} \bigtriangledown \boldsymbol{y} \wedge \mathrm{voc}(\boldsymbol{y}) \wedge \mathrm{TB}(\boldsymbol{y}) \wedge \mathrm{uvul}(\boldsymbol{y})] \\ & & \varphi_{\mathfrak{x}}(\boldsymbol{x}) := \mathrm{TB}(\boldsymbol{x}) \wedge \mathrm{phar}(\boldsymbol{x}) \wedge \mathbb{V}(\boldsymbol{x}) \\ & & \varphi_{f}(\boldsymbol{x}) := \mathrm{LIPS}(\boldsymbol{x}) \wedge \mathrm{dent}(\boldsymbol{x}) \wedge \mathrm{crit}(\boldsymbol{x}) \wedge \exists \boldsymbol{y} [\boldsymbol{x} \circlearrowright \boldsymbol{y} \wedge \mathrm{GLO}(\boldsymbol{y}) \wedge \mathrm{wide}(\boldsymbol{y})] \\ & & \varphi_{t}(\boldsymbol{x}) := \mathrm{TT}(\boldsymbol{x}) \wedge \mathrm{alv}(\boldsymbol{x}) \wedge \mathrm{clo}(\boldsymbol{x}) \wedge ((\exists \boldsymbol{y} \boldsymbol{z} [\boldsymbol{y} \lhd_{60} \boldsymbol{x} \Rightarrow (\boldsymbol{y} \circlearrowright \boldsymbol{x} \wedge \mathrm{GLO}(\boldsymbol{z}))]) \vee (\exists \boldsymbol{y} [\boldsymbol{x} \backsim \boldsymbol{y} \wedge \mathrm{GLO}(\boldsymbol{y}) \wedge \mathrm{wide}|(\boldsymbol{y}]))) \end{array}$



- Onset Cs are in phase with V and anti-phase with preceding C.
- ▶ First coda C is anti-phase with V; all other Cs eccentric with preceding C.
- $\blacktriangleright \varphi_{\lhd}(\mathbf{x}, \mathbf{y}) := (\mathbf{x} \lhd_{180} \mathbf{y}) \lor (\mathbf{x} \lhd_{30} \mathbf{y}) \lor (\mathbf{x} \diamond \mathbf{y} \land \mathbf{V}(\mathbf{y}) \land \neg \exists \mathbf{z} [\mathbf{x} \lhd_{180} \mathbf{z}])$



"spine" identification.

 $\blacktriangleright \ \varphi_{\textit{license}}(\mathbf{x}) := \neg \texttt{rel}(\mathbf{x}) \land \neg \texttt{GLO}(\mathbf{x}) \land ((\texttt{TB}(\mathbf{x}) \land \neg \texttt{V}(\mathbf{x})) \Rightarrow \neg \exists \mathbf{y} [\texttt{TT}(\mathbf{y}) \land \mathbf{x} \diamond \mathbf{y}])$





- Going from coupling graph to string removes information.
- What happens when we have to expand the representation and add more information by going from a string to a coupling graph?
- Spoiler: no real problems arise


 \triangleright C := {1, 2, 3, 4}

 Unique copy sets for primary gesture, release gesture, secondary gestures, glottal/nasal gesture

Input Workspace \triangleleft \triangleleft \triangleleft f_3 \mathbf{a}_2 l_1 t4 TT alv TB LIPS ΤT phar V dent alv clos (1,1) 2,1 (3,1) (4,1) TT rel LIPS rel TT rel (2,2) (3,2) (1,2) (4,2) $\varphi_{\text{LIPS}}^1(x) := f(x)$ $\varphi_{\text{phar}}^1 := \mathfrak{a}(x)$ TB $\varphi_{\text{LTPS}}^2(x) := \varphi_{\text{LTPS}}^1(x) \qquad \varphi_{\text{unull}}^3 := l(x)$ uvul nar $\varphi_{\mathsf{TT}}^1(x) := \mathsf{t}(x) \lor \mathsf{l}(x) \qquad \varphi_{\mathsf{clo}}^1 := \mathsf{t}(x)$ (1,3) (2,3) (3,3) (4,3) $\varphi_{\text{TT}}^2(x) := \varphi_{\text{TT}}^1(x) \qquad \varphi_{\text{crit}}^1 := f(x)$ GLO wide GLO $\varphi_{\text{TP}}^1(x) := \mathfrak{X}(x) \qquad \qquad \varphi_{\text{TP}}^1 := \mathfrak{X}(x)$ wide (1,4) (2,4) (3,4) (4,4) $\varphi_{\mathsf{TB}}^3(x) := \mathbf{l}(x)$ $\varphi_{\mathsf{nar}}^1 := \mathbf{l}(x)$ $\varphi_{\text{GLO}}^4(x) := \mathbf{t}(x) \lor \mathbf{f}(x) \qquad \varphi_{\text{nar}}^3 := \mathbf{l}(x)$ $\varphi_{\text{dent.}}^1 := f(x)$ $\varphi_{\text{wide.}}^4 := t(x) \lor f(x)$

 $\varphi^1_{alv} := t(x)$







Definition:

We note that an interpretation $K : U \to V$ gives us a construction of an internal model $\widetilde{K}(\mathcal{M})$ of U from a model M of V. We find that U and V are bi-interpretable iff, there are interpretations $K : U \to V$ and $M : V \to U$ and formulas F and G such that, for all models \mathcal{M} of V, the formula F defines an isomorphism between \mathcal{M} and $\widetilde{M}\widetilde{K}(\mathcal{M})$, and, for all models \mathcal{N} of U, the formula G defines an isomorphism between \mathcal{N} and $\widetilde{K}\widetilde{M}(\mathcal{N})$.

Friedman and Visser (2014); Oakden (2020)

Bi-Interpretability

- \mathcal{M}^s string model of *laughed*
- \mathcal{M}^{g} coupling graph model of *laughed*
- Γ^{sg} string to coupling graph transduction
- Γ^{gs} coupling graph to string transduction

$$\blacktriangleright \mathcal{M}^{s} \equiv \Gamma^{gs}(\Gamma^{sg}(\mathcal{M}^{s}))$$

- $\blacktriangleright \mathcal{M}^{g} \equiv \Gamma^{sg}(\Gamma^{gs}(\mathcal{M}^{g}))$
- ► This indicates the string and coupling graph models are **bi-interpretable**

Friedman and Visser (2014); Oakden (2020)

Conclusion

Thus we are referring to the same set of dynamically specified gestures, but this time using symbols which serve as indices to entire dynamical systems. These symbolic descriptions highlight those aspects of the gestural structures that are relevant for contrast among lexical items" (p. 241).

The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.
- ▶ Within this sphere we get bi-interpretability with string representations.

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.
- ▶ Within this sphere we get bi-interpretability with string representations.
- The string to coupling graph translation is unsurprising:

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.
- ▶ Within this sphere we get bi-interpretability with string representations.
- The string to coupling graph translation is unsurprising:

Note: while the other models represented by boxes in Figure 1 (**Coupled oscillator model of inter-gestural coordination** and **Task dynamic model of inter-articulator coordination**) are meant to be part of a model of the human speech production process, the method used for automatic generation of coupling graphs is a heuristic that is not meant to model how a speaker would go about construction a coupling graph for an arbitrary form. Coupling graphs could simply be stored by speakers in the lexicon. The automatic computation has two major benefits: (1) It represents in compact form generalizations about the set of coupling graphs that speakers use (in English, at least) and their relation to more conventional phonological representations (segments, features, syllable structure). (2) It allows the later stages of the model to be tested, by allowing automatic generation of a variety input files.

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.
- ▶ Within this sphere we get bi-interpretability with string representations.
- The string to coupling graph translation is unsurprising:

Note: while the other models represented by boxes in Figure 1 (Coupled oscillator model of inter-gestural coordination and Task dynamic model of inter-articulator coordination) are meant to be part of a model of the human speech production process, the method used for automatic generation of coupling graphs is a heuristic that is not meant to model how a speaker would go about construction a coupling graph for an arbitrary form. Coupling graphs could simply be stored by speakers in the lexicon. The automatic computation has two major benefits: (1) It represents in compact form generalizations about the set of coupling graphs that speakers use (in English, at least) and their relation to more conventional phonological representations (segments, features, syllable structure). (2) It allows the later stages of the model to be tested, by allowing automatic generation of a variety input files.

But the coupling graph to string translation is novel (as far as I'm aware).

Implement full translation for English

Implement full translation for English

▶ Provides a bridge for analyses done in AP vs. GP/string-based analyses

- Implement full translation for English
 - Provides a bridge for analyses done in AP vs. GP/string-based analyses
 - E.g. constraints over segments are easy to write but may be cumbersome for a coupling graph structure

- Implement full translation for English
 - ▶ Provides a bridge for analyses done in AP vs. GP/string-based analyses
 - E.g. constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.

- Implement full translation for English
 - ▶ Provides a bridge for analyses done in AP vs. GP/string-based analyses
 - E.g. constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
 - Sets of coupling graphs map to strings

- Implement full translation for English
 - ▶ Provides a bridge for analyses done in AP vs. GP/string-based analyses
 - E.g. constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
 - Sets of coupling graphs map to strings
 - Strings map to sets of coupling graphs

- Implement full translation for English
 - ▶ Provides a bridge for analyses done in AP vs. GP/string-based analyses
 - E.g. constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
 - Sets of coupling graphs map to strings
 - Strings map to sets of coupling graphs
 - Could explain within- and between-language variation

- Implement full translation for English
 - ▶ Provides a bridge for analyses done in AP vs. GP/string-based analyses
 - E.g. constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
 - Sets of coupling graphs map to strings
 - Strings map to sets of coupling graphs
 - Could explain within- and between-language variation
- Is string vs. coupling graph the right comparison?

- Implement full translation for English
 - ▶ Provides a bridge for analyses done in AP vs. GP/string-based analyses
 - E.g. constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
 - Sets of coupling graphs map to strings
 - Strings map to sets of coupling graphs
 - Could explain within- and between-language variation
- Is string vs. coupling graph the right comparison?
 - If the comparison is really about input-ouput mappings vs. non input-output mappings then maybe a two-level string correspondence graph is a better way to compare the two theories?

- Implement full translation for English
 - Provides a bridge for analyses done in AP vs. GP/string-based analyses
 - E.g. constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
 - Sets of coupling graphs map to strings
 - Strings map to sets of coupling graphs
 - Could explain within- and between-language variation
- Is string vs. coupling graph the right comparison?
 - If the comparison is really about input-ouput mappings vs. non input-output mappings then maybe a two-level string correspondence graph is a better way to compare the two theories?

Can we change the representations slightly to write a QF transduction?

Model theory provides a meta-language to do cross-theory comparison in phonology.

- Model theory provides a meta-language to do cross-theory comparison in phonology.
- Here, I showed that two very different theories actually represent things in quite a similar way.

- Model theory provides a meta-language to do cross-theory comparison in phonology.
- Here, I showed that two very different theories actually represent things in quite a similar way.
- Coupling graphs don't encode more information than strings, they just encode it differently.

- Model theory provides a meta-language to do cross-theory comparison in phonology.
- Here, I showed that two very different theories actually represent things in quite a similar way.
- Coupling graphs don't encode more information than strings, they just encode it differently.
- This highlights that it is not the representations that are different between the two theories, but rather how the representations are interpreted.

- Model theory provides a meta-language to do cross-theory comparison in phonology.
- Here, I showed that two very different theories actually represent things in quite a similar way.
- Coupling graphs don't encode more information than strings, they just encode it differently.
- This highlights that it is not the representations that are different between the two theories, but rather how the representations are interpreted.
 - ► AP coupling graphs already contain all the necessary phonetic information.

- Model theory provides a meta-language to do cross-theory comparison in phonology.
- Here, I showed that two very different theories actually represent things in quite a similar way.
- Coupling graphs don't encode more information than strings, they just encode it differently.
- This highlights that it is not the representations that are different between the two theories, but rather how the representations are interpreted.
 - AP coupling graphs already contain all the necessary phonetic information.
 - Strings must be further transformed somehow (but we've seen that's not too difficult to do ⁽¹⁾).

Thank You!

Bibliography I

Archangeli, D. (1988). Aspects of underspecification theory. Phonology, 5(2):183-207.

Bateman, N. (2007). A crosslinguistic investigation of palatalization. University of California, San Diego.

Browman, C. P. and Goldstein, L. (1992). Articulatory phonology: An overview. Phonetica, 49(3-4):155-180.

Browman, C. P. and Goldstein, L. (1995). Dynamics and articulatory phonology. Mind as motion, 175:193.

Browman, C. P. and Goldstein, L. M. (1986). Towards an articulatory phonology. Phonology, 3:219-252.

Chadwick, S. (2021). A Computational Analysis for Articulatory Phonology: A Duel Example with the Phonotactic Properties of Rhotic Consonants in <u>English.</u> PhD thesis, Stony Brook University.

Clements, G. N. (1985). The geometry of phonological features. Phonology, 2(1):225-252.

Courcelle, B. (1994). Monadic second-order definable graph transductions: a survey. Theoretical Computer Science, 126(1):53-75.

Davidson, L. (2004). The atoms of phonological representation: Gestures, coordination and perceptual features in consonant cluster phonotactics. The Johns Hopkins University.

Dresher, B. E. (2009). The contrastive hierarchy in phonology. Number 121. Cambridge University Press.

Engelfriet, J. and Hoogeboom, H. J. (2001). MSO definable string transductions and two-way finite-state transducers. <u>ACM Transactions on Computational</u> Logic (TOCL), 2(2):216–254.

Friedman, H. M. and Visser, A. (2014). When bi-interpretability implies synonymy. Logic Group Preprint Series, 320:1-19.

Gafos, A. I. (2002). A grammar of gestural coordination. Natural language & linguistic theory, 20(2):269-337.

Goldsmith, J. A. (1976). Autosegmental phonology. PhD thesis, Massachusetts Institute of Technology.

Goldstein, L. (2011). Back to the past tense in english. In Representing language: Essays in honor of Judith Aissen, pages 69-88.

Hall, N. E. (2003). Gestures and segments: Vowel intrusion as overlap. University of Massachusetts Amherst.

Inkelas, S. and Shih, S. S. (2016). Re-representing phonology: consequences of q theory. In Proceedings of NELS, volume 46, pages 161-174.

Jardine, A., Danis, N., and Iacoponi, L. (2021). A formal investigation of q-theory in comparison to autosegmental representations. <u>Linguistic Inquiry</u>, 52(2):333–358.

McMahon, A., Foulkes, P., and Tollfree, L. (1994). Gestural representation and lexical phonology. Phonology, 11(2):277-316.

Nam, H. (2007). A gestural coupling model of syllable structure. PhD thesis, Yale University.

Nelson, S. (2022). A model theoretic perspective on phonological feature systems. Proceedings of the Society for Computation in Linguistics, 5(1).

Oakden, C. (2020). Notational equivalence in tonal geometry. Phonology, 37(2):257-296.

Saltzman, E. (2001). Tada (task dynamics application) manual.

Strother-Garcia, K. (2019). Using model theory in phonology: a novel characterization of syllable structure and syllabification. PhD thesis, University of Delaware.

Zsiga, E. C. (1997). Features, gestures, and igbo vowels: An approach to the phonology-phonetics interface. Language, pages 227-274.