A Model Theoretic Perspective on Phonological Feature Systems

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Overview

- Model theory and logic can be used as a meta-language to compare and evaluate different types of phonological feature systems.
- Different feature systems mix $\{+, -, 0\}$ in different ways (e.g. - privative, full binary, contrastive).
- Tradeoffs between computation (logical language) and representation (primitive symbols) allow us to key in on what the meaningful differences between each system are.
- I show that if negation is used in the logical language it effectively turns everything into a full (binary) system, erasing the goals of the 0 valuations.

In particular

- **The plan:** use different logics and representational primitives to see how to formally represent three feature systems (privative, full, contrastive).
- **Diagnostic:** what natural classes do we expect versus what natural classes do certain combinations of logic/primitives predict?

Why Model Theory and Logic?

- Finite Model Theory allows for the precise definition of relational structures (e.g. - phonological strings (Libkin, 2004)).
- This method has been successful in comparing other types of phonological representations (Strother-Garcia, 2019; Jardine et al., 2021; Oakden, 2020).
- Relationship to computational complexity and learnability (Strother-Garcia et al., 2016; Vu et al., 2018; Chandlee et al., 2019).

Most importantly!

It allows for a way to quantify the differences between feature systems so that we can move beyond relying solely on our intuitions.

Phonological Features

Phonological features are present in some form in almost every modern theory of phonology and are usually traced back to the Prague school (Trubetzkoy, 1939; Jakobson et al., 1951).

- Based on phonetic properties.
- Trubetzkoy: privative, gradual, or equipollent.
  - Privative: [voice] vs []
  - Gradual: [height 1], [height 2], ... [height n]
  - Equipollent: [labial], [coronal], [dorsal]
- JFH: binary
  - Binary: [+voice] vs [-voice]

Natural Classes

- Natural classes are the result of partitioning a language's segment inventory using phonological features.
- Two traditional explanations for natural classes:

  Phonetic: All segments in a natural class share one or more phonetic property.

  Distributional: All segments in a natural class are the target/trigger for a phonological process.
- For the remainder of this talk I will assume the theory-dependent definition of natural classes from Mielke (2008):
  - A group of sounds in an inventory which share one or more distinctive features, *within a particular feature theory* to the exclusion of all other sounds in the inventory.

Interpreting Feature Bundles: Conjunction

- Feature matrices are usually interpreted as the **conjunction** of properties.
  - ...an adequate feature system should permit any natural class of sounds to be represented by the **conjunction** of features in a matrix (Kenstowicz and Kisseberth, 1979, p. 241).
  - Natural classes can be defined in terms of **conjunctions** of features... (Odden, 2005, p. 49).

Phonological interpretation

- $/\text{n}/ = \begin{bmatrix} \text{+coronal} \\ \text{+voice} \\ \text{+sonorant} \\ \text{-continuant} \\ \text{+nasal} \end{bmatrix}$
- /n/ is +coronal **AND** +voice **AND** +sonorant **AND** -continuant ...

Interpreting Feature Bundles: Zeros

- Many feature systems also include “0” notation to indicate no value for a feature.
- This is sometimes used when a feature only applies to a certain class of sounds.
  - “Trivial Underspecification” (Steriade, 1995)
- There is also “temporary underspecification” where a certain feature is not specified in the lexical representation and then filled in at the end of the phonological derivation.
  - . E.g. - voicing underspecification in sonorants (see Steriade (1995) for more discussion and review).
- Raises the question of how to formally interpret 0’s.

An aside about disjunction

- What about disjunction?
- Disjunction was allowed for triggering environments in SPE (Chomsky and Halle, 1968) using $\{\}$.
- Mielke (2008) claims that $\sim$97% of the phonologically active classes can be described with the *SPE* feature system if disjunction is allowed.
  - This is an increase of 26% from *SPE*'s coverage without disjunction.

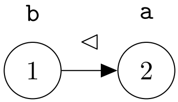
Note!

Arbitrary levels of disjunction allow any subset of segments to form a natural class.

Phonological strings with model theory

- $\mathcal{M}^{\triangleleft} = \langle \mathcal{D}, \{\mathcal{R}_\sigma | \sigma \in \Sigma\}, \triangleleft \rangle$

$$\begin{gathered} \mathcal{D} = \{1, 2\} \\ \mathcal{R}_a = \{2\} \\ \mathcal{R}_b = \{1\} \\ \triangleleft = \{\langle 1, 2 \rangle\} \end{gathered}$$

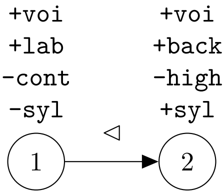


- Segmental word model for the string *ba*.
- $\Sigma = \{a, b\}$
- Can define feature values *disjunctively* using user-defined predicates, but features are not primitive. In other words, features are epiphenomenal.
- $\mathsf{voi}(x) \stackrel{\text{def}}{=} \mathcal{R}_a(x) \vee \mathcal{R}_b(x)$.

Phonological strings with model theory

- $\mathcal{M}^{\lhd} = \langle \mathcal{D}, \{\mathcal{R}_\sigma | \sigma \in \Sigma\}, \lhd \rangle$

$$\begin{aligned}
\mathcal{D} &= \{1, 2\} \\
\mathcal{R}_{\texttt{+voi}} &= \{1, 2\} \\
\mathcal{R}_{\texttt{+lab}} &= \{1\} \\
\mathcal{R}_{\texttt{-cont}} &= \{1\} \\
\mathcal{R}_{\texttt{-syl}} &= \{1\} \\
\mathcal{R}_{\texttt{+syl}} &= \{2\} \\
\mathcal{R}_{\texttt{+back}} &= \{2\} \\
\mathcal{R}_{\texttt{-high}} &= \{2\} \\
\lhd &= \{\langle 1, 2 \rangle\}
\end{aligned}$$



- Feature word model for the string *ba*.
- $\Sigma = \{\texttt{+voi,+lab,-cont,-syl,+syl,+back,-high}\}$
- Can define segments *conjunctively* using user-defined predicates because features are primitive. In other words, segments are epiphenomenal.
- $\texttt{b}(x) \overset{\text{def}}{=} \texttt{+voi}(x) \wedge \texttt{+lab}(x) \wedge \texttt{-cont}(x) \wedge \texttt{-syl}(x)$.

Formal Building Blocks

- Conjunction and a limited form of negation seem to be the only two logical connectives needed to define natural classes.
- I therefore focus on two subsets of Quantifier-Free First Order Logic.

- **Conjunction of Positive Literals (CPL)**
  - Base case: For all atoms, P, "P" is a sentence.
  - Inductive case: For all sentences A, B, "$A \wedge B$" is a sentence.
- **Conjunction of Negative and Positive Literals (CNPL)**
  - Base case: For all atoms P, "P" and "$\neg P$" are sentences.
  - Inductive case: For all sentences A, B, "$A \wedge B$" is a sentence.

Formal Building Blocks

- The atomic elements of our system will be the feature labels.
- Two sets of primitives will be considered.

- **Univalent Primitives**
  - `voi`, `son`
- **Bivalent Primitives**
  - `+voi`, `-voi`, `+son`, `-son`

A Toy Feature System

	Privative		Full		Contrastive	
	son	voice	son	voice	son	voice
N	+	+	+	+	+	0
D	0	+	-	+	-	+
T	0	0	-	-	-	-

Note: this is a slightly altered version of Table 3 in Mayer and Daland (2020).

- What are the natural classes for each system?
- **Privative:** {N}, {N,D}
- **Full:** {N}, {N,D}, {D}, {T}, {D,T}
- **Contrastive:** {N}, {D}, {T}, {D,T}

Possible combinations

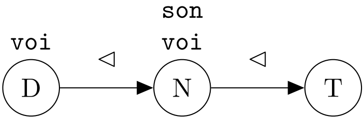
- Each set of primitives can be used to form a model signature:
  - $\mathcal{M}^{\upsilon} = \langle \mathcal{D}, \texttt{voi}, \texttt{son}, \triangleleft \rangle$
  - $\mathcal{M}^{\beta} = \langle \mathcal{D}, +\texttt{voi}, +\texttt{son}, -\texttt{voi}, -\texttt{son}, \triangleleft \rangle$
- A representation scheme can be made for each feature system (full, contrastive, binary) can be formed using each model signature, giving six potential schemes:
  - $\mathcal{M}^{\upsilon}_{\mathrm{F}}$
  - $\mathcal{M}^{\upsilon}_{\mathrm{P}}$
  - $\mathcal{M}^{\upsilon}_{\mathrm{C}}$
  - $\mathcal{M}^{\beta}_{\mathrm{F}}$
  - $\mathcal{M}^{\beta}_{\mathrm{P}}$
  - $\mathcal{M}^{\beta}_{\mathrm{C}}$
- String models based on these schemes can then be interpreted using CPL and CNPL logics to see what classes can be formed.

Interpretation

- In each case I will model the string DNT
  - Example words: bus, juice
- For the univalent models, I will assume that a + value means that domain element gets labeled with the feature.
- The graphical representation of the three models are shown on the next slide.

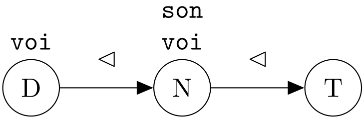
Univalent Feature Models

$\mathcal{M}^v_{\text{P}} =$



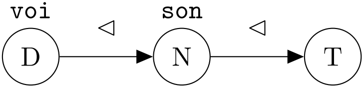
	Privative	
	son	voice
N	+	+
D	0	+
T	0	0

$\mathcal{M}^v_{\text{F}} =$



	Full	
	son	voice
N	+	+
D	-	+
T	-	-

$\mathcal{M}^c_{\text{C}} =$



	Contrastive	
	son	voice
N	+	0
D	-	+
T	-	-

CPL($\mathcal{M}^v$)

	$\mathcal{M}^v_{\mathrm{P}}$ ✓	$\mathcal{M}^v_{\mathrm{F}}$ ✗	$\mathcal{M}^v_{\mathrm{C}}$ ✗
`son`	{N}	{N}	{N}
`voi`	{N,D}	{N,D}	{D}
`son` ∧ `voi`	{N}	{N}	{}
MISSING	–	{D}, {T}, {D,T}	{T}, {D,T}
EXTRA	–	–	–

- CPL($\mathcal{M}^v_{\mathrm{P}}$) $=$ Privative
- CPL($\mathcal{M}^v_{\mathrm{F}}$) $\subsetneq$ Full
- CPL($\mathcal{M}^v_{\mathrm{C}}$) $\subsetneq$ Contrastive
- Matches Privative system but otherwise under predicts.

CNPL($\mathcal{M}^{v}$)

	$\mathcal{M}^{v}_{\text{P}}$ ✗	$\mathcal{M}^{v}_{\text{F}}$ ✓	$\mathcal{M}^{v}_{\text{C}}$ ✗
son	{N}	{N}	{N}
¬son	{D,T}	{D,T}	{D,T}
voi	{N,D}	{N,D}	{D}
¬voi	{T}	{T}	{N,T}
son ∧ ¬son	{}	{}	{}
son ∧ voi	{N}	{N}	{}
son ∧ ¬voi	{}	{}	{N}
¬son ∧ voi	{D}	{D}	{D}
¬son ∧ ¬voi	{T}	{T}	{T}
voi ∧ ¬voi	{}	{}	{}
MISSING	–	–	–
EXTRA	{D}, {T}, {D,T}	–	{N,T}

- CNPL($\mathcal{M}^{v}_{\text{P}}$) $\supsetneq$ Privative
- CNPL($\mathcal{M}^{v}_{\text{F}}$) $=$ Full
- CNPL($\mathcal{M}^{v}_{\text{C}}$) $\supsetneq$ Contrastive
- Matches Full system but otherwise over predicts.

CNPL and Equipollent Features

Features like [Labial], [Coronal] and [Dorsal] are often argued to be unary.

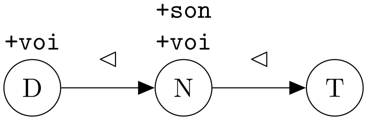
- With CNPL, if $\texttt{Coronal} \in \Sigma$ then $\neg\texttt{Coronal}$ must exist as a possible natural class.
- This example should make it clear that CNPL effectively makes all features binary.
- Note: this isn't an argument specifically about `Coronal`, but rather a more general point that *every* feature would **always** be binary.

Interpretation Redux

- For the bivalent models I will assume that a + value means that the domain element gets labeled with `+voi` or `+son` and that a − value means that the domain element gets labeled with `-voi` or `-son`.
- The graphical representation of the the three models are shown on the next slide.

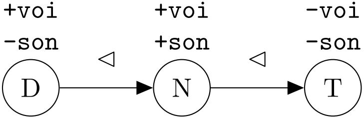
Bivalent feature models

$\mathcal{M}^{\beta}_{\mathrm{P}} =$



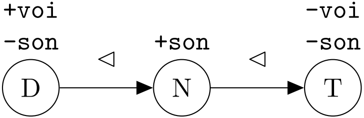
	Privative	
	son	voice
N	+	+
D	0	+
T	0	0

$\mathcal{M}^{\beta}_{\mathrm{F}} =$



	Full	
	son	voice
N	+	+
D	-	+
T	-	-

$\mathcal{M}^{\beta}_{\mathrm{C}} =$



	Contrastive	
	son	voice
N	+	0
D	-	+
T	-	-

CPL($\mathcal{M}^\beta$)

	$\mathcal{M}^\beta_\text{P}$ ✓	$\mathcal{M}^\beta_\text{F}$ ✓	$\mathcal{M}^\beta_\text{C}$ ✓
+son	{N}	{N}	{N}
-son	{}	{D,T}	{D,T}
+voi	{N,D}	{N,D}	{D}
-voi	{}	{T}	{T}
+son ∧ -son	{}	{}	{}
+son ∧ +voi	{N}	{N}	{}
+son ∧ -voi	{}	{}	{}
-son ∧ +voi	{}	{D}	{D}
-son ∧ -voi	{}	{T}	{T}
+voi ∧ -voi	{}	{}	{}

- CPL($\mathcal{M}^\beta_\text{P}$) = Privative
- CPL($\mathcal{M}^\beta_\text{F}$) = Full
- CPL($\mathcal{M}^\beta_\text{C}$) = Contrastive

Summary of CPL with Bivalent Primitives

- It can account for contrastive underspecification without creating unwanted natural classes.
- It allows for flexibility in the type of oppositions that can be encoded (binary, privative, equipollent).
- The logic on its own does not exclude an element from being both $+$`voi` and $-$`voi`?
  - Do we need to specify that we don't want this through axioms?
  - With CNPL it is impossible for an element to be both `voi` and $\neg$`voi`.

Conclusion

- Logical negation turns every feature into a binary opposition.
- Contrastive-like systems that use $\{+, -, 0\}$ require encoding the valuations directly into the primitives.
- Deciding which of these is the "right" feature system lies beyond the formal account given here.
- The findings here provide a roadmap for future work on how to best represent different types of feature systems in a formal system.
- Model theory and logic are useful for exploring formal differences between different feature systems.

Thank you!

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