What can logic and model theory tell us about phonological feature systems?

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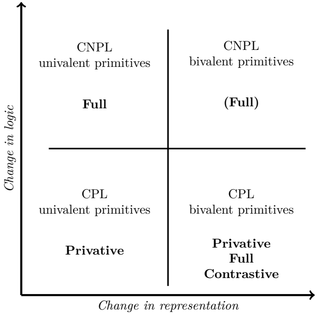
Overview

- Model theory and logic can be used as a meta-language to compare and evaluate different types of phonological feature systems.
- Different feature systems mix $\{+, -, 0\}$ in different ways (e.g. - privative, full, contrastive).
- Tradeoffs between computation (logical language) and representation (primitive symbols) allow us to key in on what the meaningful differences between each system are.
- For instance, I show that if negation is used in the logical language it effectively turns everything into a full (binary) system, erasing the goals of the 0 valuations.

In particular

- **The plan:** use different logics and representational primitives to see how to formally represent three feature systems (privative, full, contrastive).
- **Diagnostic:** what natural classes do we expect versus what natural classes do certain combinations of logic/primitives predict?



Why Model Theory and Logic?

- Finite Model Theory allows for the precise definition of relational structures (e.g. - phonological strings (Libkin, 2013)).
- This method has been successful in comparing other types of phonological representations (Strother-Garcia, 2019; Jardine et al., 2020; Oakden, 2020).
- Relationship to computational complexity and learnability (Strother-Garcia et al., 2016; Vu et al., 2018; Chandlee et al., 2019).

Most importantly!

It allows for a way to quantify the differences between feature systems so that we can move beyond relying solely on our intuitions.

Phonological Features

Phonological features are present in some form in almost every modern theory of phonology and are usually traced back to the Prague school (Trubetzkoy, 1939; Jakobson et al., 1951).

- Based on phonetic properties.
- Trubetzkoy: privative, gradual, or equipollent.
  - Privative: [voice] vs []
  - Gradual: [height 1], [height 2], ... [height n]
  - Equipollent: [labial], [coronal], [dorsal]
- JFH: binary
  - Binary: [+voice] vs [-voice]

Natural Classes

- Natural classes are the result of partitioning a language's segment inventory using phonological features.
- Two traditional explanations for natural classes:

  Phonetic: All segments in a natural class share one or more phonetic property.

  Distributional: All segments in a natural class are the target/trigger for a phonological process.
- For the remainder of this talk I will assume the theory-dependent definition of natural classes from Mielke (2008):
  - A group of sounds in an inventory which share one or more distinctive features, *within a particular feature theory* to the exclusion of all other sounds in the inventory.

Interpreting Feature Bundles: Conjunction

- Feature matrices are usually interpreted as the **conjunction** of properties.
  - ...an adequate feature system should permit any natural class of sounds to be represented by the **conjunction** of features in a matrix (Kenstowicz and Kisseberth, 1979, p. 241).
  - Natural classes can be defined in terms of **conjunctions** of features... (Odden, 2005, p. 49).

Phonological interpretation

- $/\text{n}/ = \begin{bmatrix} +\text{coronal} \\ +\text{voice} \\ +\text{sonorant} \\ \text{-continuant} \\ +\text{nasal} \end{bmatrix}$
- /n/ is +coronal **AND** +voice **AND** +sonorant **AND** -continuant ...

Interpreting Feature Bundles: Zeros

- Many feature systems also include “0” notation to indicate no value for a feature. Below is a sample from Hayes (2011).
- How do we formally interpret 0? Logical negation may lead to problems...

		Manner features								Laryngeal features			Place features													
		consonantal	sonorant	continuant	delayed release	approximant	tap	trill	nasal	voice	spread gl	constr gl	labial	round	labiodental	coronal	anterior	distributed	strident	lateral	dorsal	high	low	front	back	tense
bilabial	p	+	−	−	−	−	−	−	−	−	−	−	+	−	−	−	0	0	0	−	−	0	0	0	0	0
	b	+	−	−	−	−	−	−	−	+	−	−	+	−	−	−	0	0	0	−	−	0	0	0	0	0
	ɸ	+	−	+	+	−	−	−	−	−	−	−	+	−	−	−	0	0	0	−	−	0	0	0	0	0
	β	+	−	+	+	−	−	−	−	+	−	−	+	−	−	−	0	0	0	−	−	0	0	0	0	0
	m	+	+	−	0	−	−	−	+	+	−	−	+	−	−	−	0	0	0	−	−	0	0	0	0	0
	ʙ	+	+	+	0	+	−	+	−	+	−	−	+	−	−	−	0	0	0	−	−	0	0	0	0	0
labiodental	p͡f	+	−	−	+	−	−	−	−	−	−	−	+	−	+	−	0	0	0	−	−	0	0	0	0	0
	f	+	−	+	+	−	−	−	−	−	−	−	+	−	+	−	0	0	0	−	−	0	0	0	0	0
	v	+	−	+	+	−	−	−	−	+	−	−	+	−	+	−	0	0	0	−	−	0	0	0	0	0
	ɱ	+	+	−	0	−	−	−	+	+	−	−	+	−	+	−	0	0	0	−	−	0	0	0	0	0
	ʋ	−	+	+	0	+	−	−	−	+	−	−	+	−	+	−	0	0	0	−	−	0	0	0	0	0
dental	t̪	+	−	−	−	−	−	−	−	−	−	−	−	−	−	+	+	+	−	−	−	0	0	0	0	0
	d̪	+	−	−	−	−	−	−	−	+	−	−	−	−	−	+	+	+	−	−	−	0	0	0	0	0
	θ	+	−	+	+	−	−	−	−	−	−	−	−	−	−	+	+	+	−	−	−	0	0	0	0	0
	ð	+	−	+	+	−	−	−	−	+	−	−	−	−	−	+	+	+	−	−	−	0	0	0	0	0

An aside about disjunction

- What about disjunction?
- Disjunction was allowed for triggering environments in SPE (Chomsky and Halle, 1968) using {}.
- Mielke (2008) claims that ∼97% of the phonologically active classes can be described with the *SPE* feature system if disjunction is allowed.
  - This is an increase of 26% from *SPE*'s coverage without disjunction.

Note!

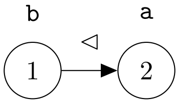
Arbitrary levels of disjunction allow any subset of segments to form a natural class.

Moving on...

Phonological strings with model theory

- $\mathcal{M}^{\lhd} = \langle \mathcal{D}, \{\mathcal{R}_\sigma | \sigma \in \Sigma\}, \lhd \rangle$

$$\begin{gathered} \mathcal{D} = \{1, 2\} \\ \mathcal{R}_a = \{2\} \\ \mathcal{R}_b = \{1\} \\ \lhd = \{\langle 1, 2 \rangle\} \end{gathered}$$

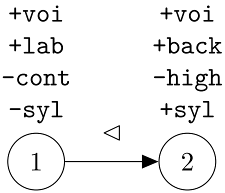


- Segmental word model for the string *ba*.
- $\Sigma = \{a, b\}$
- Can define feature values *disjunctively* using user-defined predicates, but features are not primitive. In other words, features are epiphenomenal.
- $\mathsf{voi}(x) \stackrel{\text{def}}{=} \mathcal{R}_a(x) \vee \mathcal{R}_b(x)$.

Phonological strings with model theory

- $\mathcal{M}^{\triangleleft} = \langle \mathcal{D}, \{\mathcal{R}_\sigma | \sigma \in \Sigma\}, \triangleleft \rangle$

$$\begin{aligned}
\mathcal{D} &= \{1, 2\} \\
\mathcal{R}_{\texttt{+voi}} &= \{1, 2\} \\
\mathcal{R}_{\texttt{+lab}} &= \{1\} \\
\mathcal{R}_{\texttt{-cont}} &= \{1\} \\
\mathcal{R}_{\texttt{-syl}} &= \{1\} \\
\mathcal{R}_{\texttt{+syl}} &= \{2\} \\
\mathcal{R}_{\texttt{+back}} &= \{2\} \\
\mathcal{R}_{\texttt{-high}} &= \{2\} \\
\triangleleft &= \{\langle 1, 2\rangle\}
\end{aligned}$$



- Feature word model for the string *ba*.
- $\Sigma = \{$`+voi,+lab,-cont,-syl,+syl,+back,-high`$\}$
- Can define segments *conjunctively* using user-defined predicates because features are primitive. In other words, segments are epiphenomenal.
- $\texttt{b}(x) \overset{\text{def}}{=} \texttt{+voi}(x) \wedge \texttt{+lab}(x) \wedge \texttt{-cont}(x) \wedge \texttt{-syl}(x)$.

Logical Languages

Predicate Logic
{Quantification, Disjunction, Negation, Conjunction}

$\Downarrow$

Propisitional Logic
{Disjunction, Negation, Conjunction}

$\Downarrow$

CNL/CNPL
{Conjunction, Negation only for atomic primitives}

$\Downarrow$

CPL
{Conjunction}

A Toy Feature System

	Privative		Full		Contrastive	
	son	voice	son	voice	son	voice
N	+	+	+	+	+	0
D	0	+	-	+	-	+
T	0	0	-	-	-	-

Note: this is a slightly altered version of Table 3 in Mayer and Daland (2020).

- What groups of natural classes might we expect for each system?
- **Privative:** {N}, {N,D}
- **Full:** {N}, {N,D}, {D}, {T}, {D,T}
- **Contrastive:** {N}, {D}, {T}, {D,T}

Formal Building Blocks: Logics

Logics

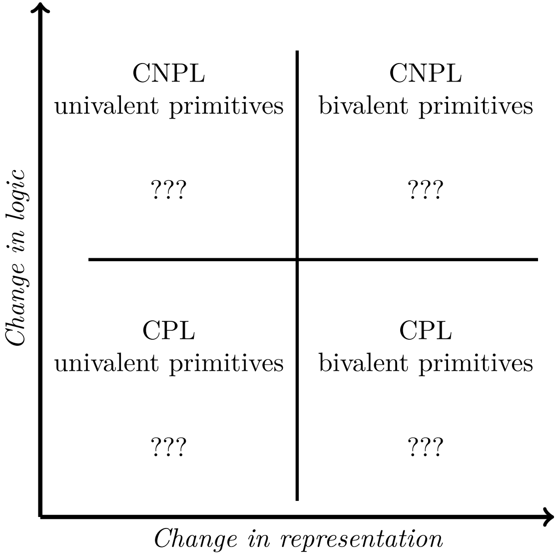
- **Conjunction of Positive Literals (CPL)**
  - Base case: For all atoms, P, "P" is a sentence.
  - Inductive case: For all sentences A, B, "$A \wedge B$" is a sentence.
- **Conjunction of Negative and Positive Literals (CNPL)**
  - Base case: For all atoms P, "P" and "$\neg P$" are sentences.
  - Inductive case: For all sentences A, B, "$A \wedge B$" is a sentence.

Formal Building Blocks: Primitives

Primitives

- **Univalent Primitives**
  - `voi`, `son`
- **Bivalent Primitives**
  - `voi`, `non-voi`, `son`, `non-son`

Possible combinations



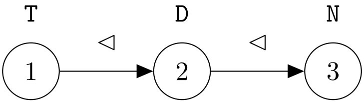
Interpretation

- We can also use model theory to translate a segmental string into one with feature values.
- This is done by defining the output string in terms of the input string: $\phi_P(x) \stackrel{\text{def}}{=} Q(x)$.
  - Read as: "Domain element x has property P in the output if it had property Q in the input."
  - P and Q are logical statements.
  - See appendix if interested.
- As a first approximation I will assume the features are $\{\texttt{voi}, \texttt{son}\}$ and a $+$ value means that domain element gets labeled with the feature.
- Three translations: $\mathcal{T}_{\mathcal{P}}, \mathcal{T}_{\mathcal{F}}, \mathcal{T}_{\mathcal{C}}$

Basic Segmental Model

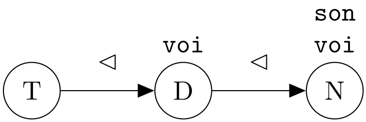
- Successor model with segments for string TDN: $\mathcal{M}^{\lhd}_{\mathtt{TDN}}$

$$
\begin{aligned}
\mathcal{D} &= \{1, 2, 3\} \\
\mathcal{R}_{\mathtt{T}} &= \{1\} \\
\mathcal{R}_{\mathtt{D}} &= \{2\} \\
\mathcal{R}_{\mathtt{N}} &= \{3\} \\
\lhd &= \{\langle 1, 2\rangle \, \langle 2, 3\rangle\}
\end{aligned}
$$



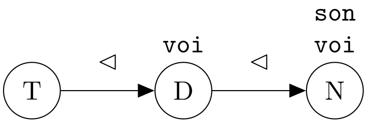
Translations from segment model to univalent feature model

$$\mathcal{T}_{\mathcal{P}}(\mathcal{M}^{\lhd}_{\texttt{TDN}}) =$$



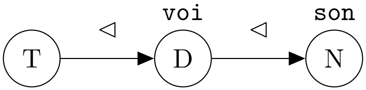
	Privative	
	son	voice
N	+	+
D	0	+
T	0	0

$$\mathcal{T}_{\mathcal{F}}(\mathcal{M}^{\lhd}_{\texttt{TDN}}) =$$



	Full	
	son	voice
N	+	+
D	-	+
T	-	-

$$\mathcal{T}_{\mathcal{C}}(\mathcal{M}^{\lhd}_{\texttt{TDN}}) =$$

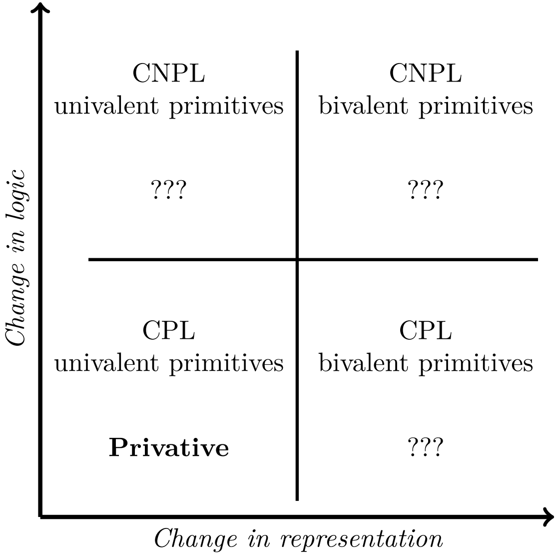


	Contrastive	
	son	voice
N	+	0
D	-	+
T	-	-

CPL(son,voi)

	Privative ✓	Full ✗	Contrastive ✗
son	{N}	{N}	{N}
voi	{N,D}	{N,D}	{D}
son $\wedge$ voi	{N}	{N}	{}
Missing	–	{D}, {T}, {D,T}	{T}, {D,T}
Extra	–	–	–

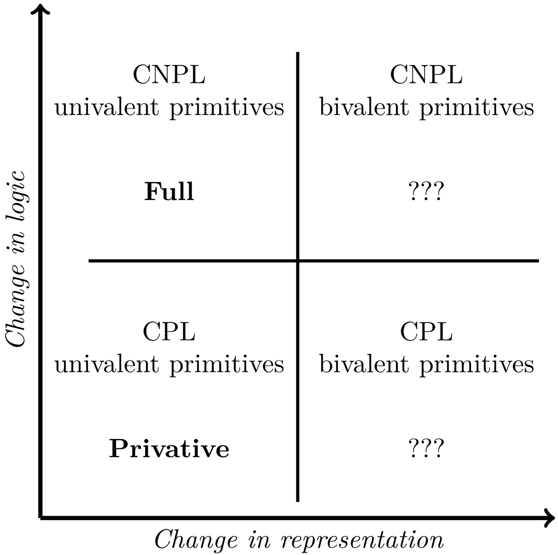
- $\mathrm{CPL}(\mathcal{T}_{\mathcal{P}}) =$ Privative
- $\mathrm{CPL}(\mathcal{T}_{\mathcal{F}}) \subsetneq$ Full
- $\mathrm{CPL}(\mathcal{T}_{\mathcal{C}}) \subsetneq$ Contrastive
- Matches Privative system but otherwise under predicts.


CNPL
univalent primitives
???
CNPL
bivalent primitives
???
CPL
univalent primitives
Privative
CPL
bivalent primitives
???
Change in logic
Change in representation

CNPL(voi,son)

	Privative ✗	Full ✓	Contrastive ✗
son	{N}	{N}	{N}
¬son	{D,T}	{D,T}	{D,T}
voi	{N,D}	{N,D}	{D}
¬voi	{T}	{T}	{N,T}
son ∧ ¬son	{}	{}	{}
son ∧ voi	{N}	{N}	{}
son ∧ ¬voi	{}	{}	{N}
¬son ∧ voi	{D}	{D}	{D}
¬son ∧ ¬voi	{T}	{T}	{T}
voi ∧ ¬voi	{}	{}	{}
Missing	–	–	–
Extra	{D}, {T}, {D,T}	–	{N,T}

- $\mathrm{CNPL}(\mathcal{T}_{\mathcal{P}}) \supsetneq$ Privative
- $\mathrm{CNPL}(\mathcal{T}_{\mathcal{F}}) =$ Full
- $\mathrm{CNPL}(\mathcal{T}_{\mathcal{C}}) \supsetneq$ Contrastive
- Matches Full system but otherwise over predicts.


CNPL
univalent primitives
Full
CNPL
bivalent primitives
???
CPL
univalent primitives
Privative
CPL
bivalent primitives
???
Change in logic
Change in representation

CNPL and Equipollent Features

Features like [Labial], [Coronal] and [Dorsal] are often argued to be unary.

- With CNPL, if `Coronal` $\in \Sigma$ then $\neg$`Coronal` must exist as a possible natural class.
- This example should make it clear that CNPL effectively makes all features binary.
- Note: this isn't an argument specifically about `Coronal`, but rather a more general point that *every* feature would **always** be binary.

Interpretation Redux

- We need updated feature translations if we change our primitives to include both positive and negative features.
- As a second approximation I will assume the features are {`voi`, `son`, `non-voi`, `non-son`}
- A + value means that the domain element gets labeled with `voi` or `son`.
- A − value means that the domain element gets labeled with `non-voi` or `non-son`.
- Three translations: $\mathcal{T}_{\mathcal{P}'}, \mathcal{T}_{\mathcal{F}'}, \mathcal{T}_{\mathcal{C}'}$

Translations from segment model to bivalent feature model

$$\mathcal{T}_{\mathcal{P}'}(\mathcal{M}^{\lhd}_{\mathtt{TDN}}) = \begin{array}{ccccc} & & & & \mathtt{son} \\ & & \mathtt{voi} & & \mathtt{voi} \\ \mathrm{T} & \xrightarrow{\lhd} & \mathrm{D} & \xrightarrow{\lhd} & \mathrm{N} \end{array}$$

	Privative	
	son	voice
N	+	+
D	0	+
T	0	0

$$\mathcal{T}_{\mathcal{F}'}(\mathcal{M}^{\lhd}_{\mathtt{TDN}}) = \begin{array}{ccccc} \mathtt{non\text{-}voi} & & \mathtt{voi} & & \mathtt{voi} \\ \mathtt{non\text{-}son} & & \mathtt{non\text{-}son} & & \mathtt{son} \\ \mathrm{T} & \xrightarrow{\lhd} & \mathrm{D} & \xrightarrow{\lhd} & \mathrm{N} \end{array}$$

	Full	
	son	voice
N	+	+
D	-	+
T	-	-

$$\mathcal{T}_{\mathcal{C}'}(\mathcal{M}^{\lhd}_{\mathtt{TDN}}) = \begin{array}{ccccc} \mathtt{non\text{-}voi} & & \mathtt{voi} & & \\ \mathtt{non\text{-}son} & & \mathtt{non\text{-}son} & & \mathtt{son} \\ \mathrm{T} & \xrightarrow{\lhd} & \mathrm{D} & \xrightarrow{\lhd} & \mathrm{N} \end{array}$$

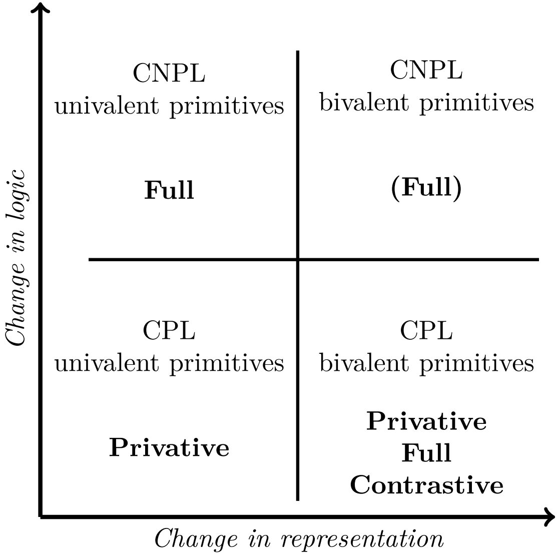
	Contrastive	
	son	voice
N	+	0
D	-	+
T	-	-

CPL(`voi`,`non-voi`,`son`,`non-son`)

	Privative ✓	Full ✓	Contrastive ✓
`son`	{N}	{N}	{N}
`non-son`	{}	{D,T}	{D,T}
`voi`	{N,D}	{N,D}	{D}
`non-voi`	{}	{T}	{T}
`son` ∧ `non-son`	{}	{}	{}
`son` ∧ `voi`	{N}	{N}	{}
`son` ∧ `non-voi`	{}	{}	{}
`non-son` ∧ `voi`	{}	{D}	{D}
`non-son` ∧ `non-voi`	{}	{T}	{T}
`voi` ∧ `non-voi`	{}	{}	{}

- $\mathrm{CPL}(\mathcal{T}_{\mathcal{P}'}) =$ Privative
- $\mathrm{CPL}(\mathcal{T}_{\mathcal{F}'}) =$ Full
- $\mathrm{CPL}(\mathcal{T}_{\mathcal{C}'}) =$ Contrastive

Questions going forward...



Summary of CPL with Bivalent Primitives

- It can account for contrastive underspecification without creating unwanted natural classes.
- It allows for flexibility in the type of oppositions that can be encoded (binary, privative, equipollent).
- The logic on its own does not exclude an element from being both `voice` and `non-voice`?
  - Do we need to specify that we don't want this through axioms?
  - With CNPL it is impossible for an element to be both `voice` and ¬`voice`.

Conclusion

- Model theory and logic are useful for exploring formal differences between different feature systems.
- Logical negation turns every feature into a binary opposition.
- Contrastive-like systems that use $\{+, -, 0\}$ require encoding the valuations directly into the primitives.

Thank you!

- A special thank you goes out to Jeffrey Heinz, Karthik Durvasula, Nick Danis, Charles Reiss, Eric Baković, and the Stony Brook/Rutgers Spring 2021 MathLing Reading Group for helpful comments and discussion on this material.

Bibliography

Chandlee, J., Eyraud, R., Heinz, J., Jardine, A., and Rawski, J. (2019). Learning with partially ordered representations. In Proceedings of the 16th Meeting on the Mathematics of Language, pages 91–101, Toronto, Canada. Association for Computational Linguistics.

Chomsky, N. and Halle, M. (1968). The sound pattern of English. Harper & Row.

Hayes, B. (2011). Introductory phonology, volume 32. John Wiley & Sons.

Jakobson, R., Fant, C. G., and Halle, M. (1951). Preliminaries to speech analysis: The distinctive features and their correlates. MIT press.

Jardine, A., Danis, N., and Iacoponi, L. (2020). A formal investigation of q-theory in comparison to autosegmental representations. Linguistic Inquiry, pages 1–25.

Kenstowicz, M. and Kisseberth, C. (1979). Generative phonology: Description and theory. Academic Press.

Libkin, L. (2013). Elements of finite model theory. Springer Science & Business Media.

Mayer, C. and Daland, R. (2020). A method for projecting features from observed sets of phonological classes. Linguistic Inquiry, 51(4):725–763.

Mielke, J. (2008). The emergence of distinctive features. Oxford University Press.

Oakden, C. (2020). Notational equivalence in tonal geometry. Phonology, 37(2).

Odden, D. (2005). Introducing phonology. Cambridge university press.

Strother-Garcia, K. (2019). Using Model Theory in Phonology: A Novel Characterization of Syllable Structure and Syllabification. PhD thesis, University of Delaware.

Strother-Garcia, K., Heinz, J., and Hwangbo, H. J. (2016). Using model theory for grammatical inference: a case study from phonology. In Verwer, S., van Zaanen, M., and Smetsers, R., editors, Proceedings of The 13th International Conference on Grammatical Inference, volume 57 of JMLR: Workshop and Conference Proceedings, pages 66–78.

Trubetzkoy, N. S. (1939). Principles of phonology. ERIC.

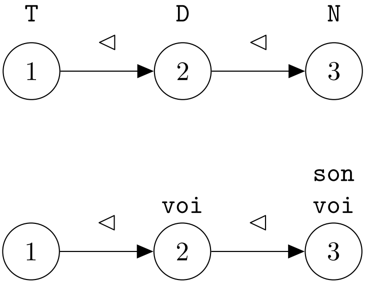
Vu, M. H., Zehfroosh, A., Strother-Garcia, K., Sebok, M., Heinz, J., and Tanner, H. G. (2018). Statistical relational learning with unconventional string models. Frontiers in Robotics and AI, 5(76):1–26.

Translation into feature model (Primitive + Full)

- $\mathcal{T}_{\mathcal{P}} = \mathcal{T}_{\mathcal{F}}$

$$\phi_{\lhd}(x, y) \stackrel{\text{def}}{=} \lhd(x, y)$$
$$\phi_{\texttt{son}}(x) \stackrel{\text{def}}{=} \texttt{N}(x)$$
$$\phi_{\texttt{voi}}(x) \stackrel{\text{def}}{=} \texttt{D}(x) \vee \texttt{N}(x)$$

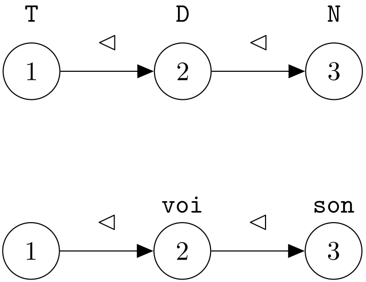
- Bottom graph shows $\mathcal{T}_{\mathcal{P}}(\mathcal{M}^{\lhd}_{\texttt{TDN}}) = \mathcal{T}_{\mathcal{F}}(\mathcal{M}^{\lhd}_{\texttt{TDN}})$



Translation into Feature Model (Contrastive)

- $\mathcal{T}_{\mathcal{C}}$

$$\begin{aligned}
\phi_{\lhd}(x,y) &\stackrel{\text{def}}{=} \lhd(x,y)\\
\phi_{\texttt{son}}(x) &\stackrel{\text{def}}{=} \texttt{N}(x)\\
\phi_{\texttt{voi}}(x) &\stackrel{\text{def}}{=} \texttt{D}(x)
\end{aligned}$$

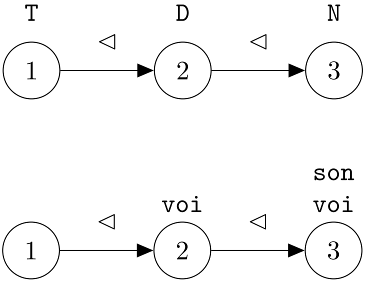


- Bottom graph shows $\mathcal{T}_{\mathcal{C}}(\mathcal{M}^{\lhd}_{\texttt{TDN}})$

Translation into feature model (Primitive) redux

- $\mathcal{T}_{\mathcal{P}'}$

$$
\begin{aligned}
\phi_{\lhd}(x, y) &\stackrel{\text{def}}{=} \lhd(x, y)\\
\phi_{\texttt{son}}(x) &\stackrel{\text{def}}{=} \texttt{N}(x)\\
\phi_{\texttt{voi}}(x) &\stackrel{\text{def}}{=} \texttt{D}(x) \lor \texttt{N}(x)\\
\phi_{\texttt{non-son}}(x) &\stackrel{\text{def}}{=} \texttt{false}\\
\phi_{\texttt{non-voi}}(x) &\stackrel{\text{def}}{=} \texttt{false}
\end{aligned}
$$



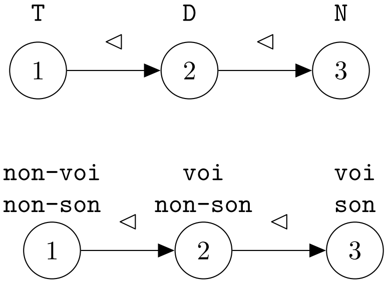
- Bottom graph shows $\mathcal{T}_{\mathcal{P}'}(\mathcal{M}^{\lhd}_{\texttt{TDN}})$

Translation into feature model (Full) redux

- $\mathcal{T}_{\mathcal{F}'}$

$$\phi_{\lhd}(x, y) \stackrel{\text{def}}{=} \lhd(x, y)$$
$$\phi_{\texttt{son}}(x) \stackrel{\text{def}}{=} \texttt{N}(x)$$
$$\phi_{\texttt{voi}}(x) \stackrel{\text{def}}{=} \texttt{D}(x) \vee \texttt{N}(x)$$
$$\phi_{\texttt{non-son}}(x) \stackrel{\text{def}}{=} \texttt{D}(x) \vee \texttt{T}(x)$$
$$\phi_{\texttt{non-voi}}(x) \stackrel{\text{def}}{=} \texttt{T}(x)$$

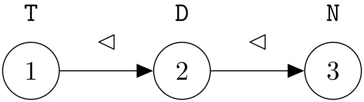
- Bottom graph shows $\mathcal{T}_{\mathcal{F}'}(\mathcal{M}^{\lhd}_{\texttt{TDN}})$

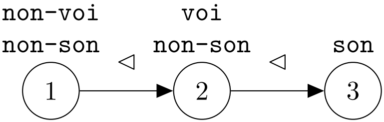


Translation into Feature Model (Contrastive) redux

- $\mathcal{T}_{\mathcal{C}'}$

$$\phi_{\triangleleft}(x,y) \stackrel{\text{def}}{=} \triangleleft(x,y)$$
$$\phi_{\texttt{son}}(x) \stackrel{\text{def}}{=} \texttt{N}(x)$$
$$\phi_{\texttt{voi}}(x) \stackrel{\text{def}}{=} \texttt{D}(x)$$
$$\phi_{\texttt{non-son}}(x) \stackrel{\text{def}}{=} \texttt{D}(x) \vee \texttt{T}(x)$$
$$\phi_{\texttt{non-voi}}(x) \stackrel{\text{def}}{=} \texttt{T}(x)$$





- Bottom graph shows $\mathcal{T}_{\mathcal{C}'}(\mathcal{M}^{\triangleleft}_{\texttt{TDN}})$