The logical language of phonological features

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Introduction

- Phonological features in some form are present in almost every modern theory of phonology.
- It is rare to find a formal definition of phonological features and how they combine (see Bale and Reiss (2018) for one example).
- What can we learn by using logic to explore feature systems?
- What are the minimally necessary aspect of features systems that give phonologists what they want out of them?

Preliminaries

The goal of this talk is to investigate phonological feature systems in order to better understand their computational properties. Because of this...

- I will assume features are discrete categories.
- I will make no claim on whether features are innate or emergent.
- I will make no claim on how much and/or what type of phonetic substance is found in features.

Phonological Features

The origin of distinctive feature theory is traced back to the Prague school (Trubetzkoy, 1939; Jakobson et al., 1951).

- Based on phonetic properties.
- Trubetzkoy: features can be privative, gradual, or equipollent.
- JFH: binary features only.

Phonological Features

- Privative: [voice] vs []
- Gradual: [height 1], [height 2], ... [height n]
- equipollent: [labial], [coronal], [dorsal]
- Binary: [+voice] vs [-voice]

Natural Classes

Natural classes can be defined in two ways (Mielke, 2008).

- A group of sounds in an inventory which share one or more distinctive features, to the exclusion of all other sounds in the inventory.
- A group of sounds in an inventory which may participate in an alternation or static distributional restriction, to the exclusion of all other sounds in the inventory.

The Use of Zeros

Many feature systems include “0” notation to indicate no value for a feature. Below is a sample from Hayes (2011).

		Manner features								Laryngeal features			Place features													
		consonantal	sonorant	continuant	delayed release	approximant	tap	trill	nasal	voice	spread gl	constr gl	labial	round	labiodental	coronal	anterior	distributed	strident	lateral	dorsal	high	low	front	back	tense
bilabial	p	+	–	–	–	–	–	–	–	–	–	–	+	–	–	–	0	0	0	–	–	0	0	0	0	0
	b	+	–	–	–	–	–	–	–	+	–	–	+	–	–	–	0	0	0	–	–	0	0	0	0	0
	ɸ	+	–	+	+	–	–	–	–	–	–	–	+	–	–	–	0	0	0	–	–	0	0	0	0	0
	β	+	–	+	+	–	–	–	–	+	–	–	+	–	–	–	0	0	0	–	–	0	0	0	0	0
	m	+	+	–	0	–	–	–	+	+	–	–	+	–	–	–	0	0	0	–	–	0	0	0	0	0
	ʙ	+	+	+	0	+	–	+	–	+	–	–	+	–	–	–	0	0	0	–	–	0	0	0	0	0
labiodental	p͡f	+	–	–	+	–	–	–	–	–	–	–	+	–	+	–	0	0	0	–	–	0	0	0	0	0
	f	+	–	+	+	–	–	–	–	–	–	–	+	–	+	–	0	0	0	–	–	0	0	0	0	0
	v	+	–	+	+	–	–	–	–	+	–	–	+	–	+	–	0	0	0	–	–	0	0	0	0	0
	ɱ	+	+	–	0	–	–	–	+	+	–	–	+	–	+	–	0	0	0	–	–	0	0	0	0	0
	ʋ	–	+	+	0	+	–	–	–	+	–	–	+	–	+	–	0	0	0	–	–	0	0	0	0	0
dental	t̪	+	–	–	–	–	–	–	–	–	–	–	–	–	–	+	+	+	–	–	–	0	0	0	0	0
	d̪	+	–	–	–	–	–	–	–	+	–	–	–	–	–	+	+	+	–	–	–	0	0	0	0	0
	θ	+	–	+	+	–	–	–	–	–	–	–	–	–	–	+	+	+	–	–	–	0	0	0	0	0
	ð	+	–	+	+	–	–	–	–	+	–	–	–	–	–	+	+	+	–	–	–	0	0	0	0	0

Interpreting Feature Bundles

Feature matrices are interpreted as the **conjunction** of properties.

$$/\text{n}/ = \begin{bmatrix} +\text{alveolar} \\ +\text{voice} \\ +\text{sonorant} \\ +\text{continuant} \\ +\text{nasal} \end{bmatrix}$$

Disjunction was allowed for triggering environments in SPE (Chomsky and Halle, 1968) using {}.

Note!

Arbitrary levels of disjunction allow any subset of segments to be natural classes.

Logical Languages

What does our logic need?

The strategy should be start with the simplest logic and only go to higher levels only if necessary.

MSO/FO Logic
{Quantification, Disjunction, Negation, Conjunction}

$\Downarrow$

Predicate Logic
{Disjunction, Negation, Conjunction}

$\Downarrow$

CNL/CNPL
{Conjunction, Negation only for atomic primitives}

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CPL
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Conjunction of Positive Literals

- Base case: For all atoms P, "P" is a sentence.
- Inductive case: For all sentences A, B, "$A \wedge B$" is a sentence.

A Toy Feature System

	Privative		Full		Contrastive	
	son	voice	son	voice	son	voice
N	+	+	+	+	+	0
D	0	+	-	+	-	+
T	0	0	-	-	-	-

Note: this is a slightly altered version of Table 3 in Mayer and Daland (2020).

CPL and *NT

Given $\Sigma = \{\texttt{voice}, \texttt{sonorant}\}$, CPL cannot pick out the natural class of voiceless stops required for the constraint *NT. It defaults to the privative feature system.

- $\texttt{voice}(x) = \{N, D\}$
- $\texttt{sonorant}(x) = \{N\}$
- $\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{N\}$

	Privative	
	son	voice
N	+	+
D	0	+
T	0	0

Including "Minus" Features

There are two ways that we can add a "minus" feature value to the logic.

- Option 1: Add negation to the logic (CNPL)
  - Only to base case...adding to inductive case gets us predicate logic.
- Option 2: Use bivalent primitives (remain in CPL).
  - E.g., `voice` and `non-voice` are both atomic elements.

Conjunction of Negative and Positive Literals

- Base case: For all atoms P, "P" and "$\neg P$" are sentences.
- Inductive case: For all sentences A, B, "$A \wedge B$" is a sentence.

CNPL and *NT

Given $\Sigma = \{\texttt{voice}, \texttt{sonorant}\}$, CNPL can pick out the natural class of voiceless stops required for the constraint *NT.

- $\texttt{voice}(x) = \{N, D\}$
- $\neg\texttt{voice}(x) = \{T\}$
- $\texttt{sonorant}(x) = \{N\}$
- $\neg\texttt{sonorant}(x) = \{D, T\}$
- $\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{N\}$
- $\texttt{voice}(x) \wedge \neg\texttt{sonorant}(x) = \{D\}$
- $\neg\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{\}$
- $\neg\texttt{voice}(x) \wedge \neg\texttt{sonorant}(x) = \{T\}$
- $\texttt{voice}(x) \wedge \neg\texttt{voice}(x) = \{\}$
- $\texttt{sonorant}(x) \wedge \neg\texttt{sonorant}(x) = \{\}$

	Full	
	son	voice
N	+	+
D	-	+
T	-	-

CNPL and *NT

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- $\texttt{voice}(x) = \{N, D\}$
- $\boxed{\neg\texttt{voice}(x) = \{T\}}$
- $\texttt{sonorant}(x) = \{N\}$
- $\neg\texttt{sonorant}(x) = \{D, T\}$
- $\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{N\}$
- $\texttt{voice}(x) \wedge \neg\texttt{sonorant}(x) = \{D\}$
- $\neg\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{\}$
- $\boxed{\neg\texttt{voice}(x) \wedge \neg\texttt{sonorant}(x) = \{T\}}$
- $\texttt{voice}(x) \wedge \neg\texttt{voice}(x) = \{\}$
- $\texttt{sonorant}(x) \wedge \neg\texttt{sonorant}(x) = \{\}$

	Full	
	son	voice
N	+	+
D	-	+
T	-	-

CNPL and Underspecification

Given $\Sigma = \{\texttt{voice}, \texttt{sonorant}\}$, and contrastive underspecification for voice, CNPL predicts voiceless stops and nasals to form a natural class.

- $\texttt{voice}(x) = \{D\}$
- $\neg\texttt{voice}(x) = \{N, T\}$
- $\texttt{sonorant}(x) = \{N\}$
- $\neg\texttt{sonorant}(x) = \{D, T\}$
- $\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{\}$
- $\texttt{voice}(x) \wedge \neg\texttt{sonorant}(x) = \{D\}$
- $\neg\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{N\}$
- $\neg\texttt{voice}(x) \wedge \neg\texttt{sonorant}(x) = \{T\}$
- $\texttt{voice}(x) \wedge \neg\texttt{voice}(x) = \{\}$
- $\texttt{sonorant}(x) \wedge \neg\texttt{sonorant}(x) = \{\}$

	Contrastive	
	son	voice
N	+	0
D	-	+
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- $\boxed{\neg\texttt{voice}(x) = \{N, T\}}$
- $\texttt{sonorant}(x) = \{N\}$
- $\neg\texttt{sonorant}(x) = \{D, T\}$
- $\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{\}$
- $\texttt{voice}(x) \wedge \neg\texttt{sonorant}(x) = \{D\}$
- $\neg\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{N\}$
- $\neg\texttt{voice}(x) \wedge \neg\texttt{sonorant}(x) = \{T\}$
- $\texttt{voice}(x) \wedge \neg\texttt{voice}(x) = \{\}$
- $\texttt{sonorant}(x) \wedge \neg\texttt{sonorant}(x) = \{\}$

	Contrastive	
	son	voice
N	+	0
D	-	+
T	-	-

CNPL and Equipollent Features

Recall that features like [Labial], [Coronal] and [Dorsal] are argued to be equipollent.

- In CNPL, if `Coronal` $\in \Sigma$ then $\neg$`Coronal` must exist as a possible natural class.
- As Yip (1989) argues, this natural class is not found in natural language.
- This example should make it clear that CNPL effectively makes all features binary.

Interim Summary

- CPL with privative primitives cannot directly reference “minus” features such as voicelessness.
- CNPL can reference “minus” features.
- CNPL does not treat underspecification properly.
- CNPL turns every feature into a binary feature.
- Let’s now turn our attention to CPL with bivalent primitives.

Bivalent Primitives

- Bivalent primitives encode the traditional idea of binary features without using logical negation.
- I will use the prefix `non-` to indicate the negative value of a feature.
- E.g. `voice` and `non-voice`.

CPL with Bivalent Primitives and *NT

Given $\Sigma = \{\texttt{voice}, \texttt{non-voice}, \texttt{sonorant}, \texttt{non-sonorant}\}$, CPL can pick out the natural class of voiceless stops required for the constraint *NT.

- $\texttt{voice}(x) = \{N, D\}$
- $\texttt{non-voice}(x) = \{T\}$
- $\texttt{sonorant}(x) = \{N\}$
- $\texttt{non-sonorant}(x) = \{D, T\}$
- $\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{N\}$
- $\texttt{voice}(x) \wedge \texttt{non-sonorant}(x) = \{D\}$
- $\texttt{non-voice}(x) \wedge \texttt{sonorant}(x) = \{\}$
- $\texttt{non-voice}(x) \wedge \texttt{non-sonorant}(x) = \{T\}$
- $\texttt{voice}(x) \wedge \texttt{non-voice}(x) = \{\}$
- $\texttt{sonorant}(x) \wedge \texttt{non-sonorant}(x) = \{\}$

	Full	
	son	voice
N	+	+
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Given $\Sigma = \{\texttt{voice}, \texttt{non-voice}, \texttt{sonorant}, \texttt{non-sonorant}\}$, CPL can pick out the natural class of voiceless stops required for the constraint *NT.

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- $\texttt{non-voice}(x) \wedge \texttt{sonorant}(x) = \{\}$
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- $\texttt{voice}(x) \wedge \texttt{non-voice}(x) = \{\}$
- $\texttt{sonorant}(x) \wedge \texttt{non-sonorant}(x) = \{\}$

	Full	
	son	voice
N	+	+
D	-	+
T	-	-

CPL with Bivalent Primitives and Underspecification

Given $\Sigma = \{\texttt{voice}, \texttt{non-voice}, \texttt{sonorant}, \texttt{non-sonorant}\}$, and contrastive underspecification for voice, CPL does not create any undesirable natural classes.

- $\texttt{voice}(x) = \{D\}$
- $\texttt{non-voice}(x) = \{T\}$
- $\texttt{sonorant}(x) = \{N\}$
- $\texttt{non-sonorant}(x) = \{D, T\}$
- $\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{\}$
- $\texttt{voice}(x) \wedge \texttt{non-sonorant}(x) = \{D\}$
- $\texttt{non-voice}(x) \wedge \texttt{sonorant}(x) = \{N\}$
- $\texttt{non-voice}(x) \wedge \texttt{non-sonorant}(x) = \{T\}$
- $\texttt{voice}(x) \wedge \texttt{non-voice}(x) = \{\}$
- $\texttt{sonorant}(x) \wedge \texttt{non-sonorant}(x) = \{\}$

	Contrastive	
	son	voice
N	+	0
D	-	+
T	-	-

CPL with Bivalent Primitives and Underspecification

Given $\Sigma = \{\texttt{voice}, \texttt{non-voice}, \texttt{sonorant}, \texttt{non-sonorant}\}$, and privative underspecification, CPL creates the same natural classes as CPL with monovalent primitives.

- $\texttt{voice}(x) = \{D, N\}$
- $\texttt{non-voice}(x) = \{\}$
- $\texttt{sonorant}(x) = \{N\}$
- $\texttt{non-sonorant}(x) = \{\}$
- $\texttt{voice}(x) \wedge \texttt{sonorant}(x) = \{N\}$
- $\texttt{voice}(x) \wedge \texttt{non-sonorant}(x) = \{\}$
- $\texttt{non-voice}(x) \wedge \texttt{sonorant}(x) = \{\}$
- $\texttt{non-voice}(x) \wedge \texttt{non-sonorant}(x) = \{\}$
- $\texttt{voice}(x) \wedge \texttt{non-voice}(x) = \{\}$
- $\texttt{sonorant}(x) \wedge \texttt{non-sonorant}(x) = \{\}$

	Privative	
	son	voice
N	+	+
D	0	+
T	0	0

CPL with Bivalent Primitives and Equipollent Features

Let's return to the equipollent features [Labial], [Coronal] and [Dorsal].

- In CPL, $\texttt{Coronal} \in \Sigma \not\rightarrow \texttt{non-Coronal} \in \Sigma$.
- CPL therefore correctly encodes the idea of equipollency.
- With CPL, binarity does not emerge from the logical language the same way it does in CNPL.

Summary of CPL with Bivalent Primitives

- It can account for "minus" feature natural classes.
- It can account for underspecification without creating unwanted natural classes.
- It allows for flexibility in the type of oppositions that can be encoded (binary, privative, equipollent).

Potential Problems Involving CPL with Bivalent Primitives

- In CNPL it is impossible for an element to be both `voice` and ¬`voice`.
- What does it mean for an element to be both `voice` and `non-voice`?
  - Do we need to specify that we don't want this through axioms?
- If features = instructions for phonetic implementation then this might not be a problem.
  - Evidence might come in the form of gesture blending (Zsiga et al., 1995; Honorof, 2000).
  - Mid vowels could be **`high`** and **`non-high`** where the blended form would be in between the two.

Conclusion

- There are two ways to handle minus features.
  - Enrich the logic: CNPL.
  - Enrich the representations: bivalent primitives.
- Both options come with consequences.
  - CNPL turns every feature into a binary opposition.
  - Bivalent primitives allow elements to have both positive and negative values for a feature.
- If we want to have non-binary phonological features, then CPL with bivalent primitives appears to be the way to go.

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