Phonological Knowledge, Weighted Logic, and the Competence/Performance Distinction

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Optional Theme: Phono–Logical Reasoning

NAPhC seeks submissions that contribute to the understanding of the phonological component of the human language faculty. We particularly welcome papers on the relationship of formal reasoning to I-language phonology in (at least one of) the three following interrelated senses:

1. The logic$_1$ of phonological computation and representation: Do rules/constraints make explicit use of quantificational logic and set theoretic notions? What is the logical structure of segments and other representational units?
2. The logic$_2$ of phonological acquisition: What logical apparatus must be used by the Phonological Language Acquisition Device to discover the rules and representations that end up in long term memory?
3. The logic$_3$ of phonology-the-discipline: What kinds of reasoning do phonologists use to figure out the contents of logic$_1$ and logic$_2$

For example, one might argue that logic$_2$ and logic$_3$ make reference to meaning contrasts in the lexicon, say, by comparing minimal pairs, but this does not entail that logic$_1$ makes reference to contrast, for example, by avoiding derivations that result in homophony, or treating non-contrastive features as inert. Of course, some scholars have argued that logic$_1$ *does* make reference to contrast, and such questions are open to discussion.

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Phonological Reasoning

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3. The logic_3 of phonology-the-discipline: W

Phonological Knowledge

- Broadly speaking, there are two types of synchronic knowledge we may be interested in as phonologists.

1. Representational Knowledge
   - What is a possible long term memory representation?
   - What is a possible output of the phonological grammar?
2. Process Knowledge
   - How do we relate these two types of representations?

cf. Anderson (1985); Heinz (2018)

Phonological Knowledge

- Some researchers now believe that phonological knowledge should also include the rate at which phonological generalizations hold.

- This is due to experimental data showing gradient behavior which has lead to **The law of frequency matching**:

> "speakers of languages with variable lexical patterns respond stochastically when tested on such patterns. Their responses aggregately match the lexical frequencies."

Hayes et al. (2009, p. 826)

Nasal-place assimilation

- In English, coronal nasals assimilate in place to a following obstruent across word boundaries.
- The status of non-coronal nasal assimilation is contested, but it has been argued to be absent in certain dialects.

- *i*[m] [$\mathrm{p^h}$]*ort Jefferson*
- *i*[ŋ] [$\mathrm{k^h}$]*anada*
- *i*[n] [$\mathrm{t^h}$]*acoma*

- *fro*[m] [$\mathrm{p^h}$]*ort Jefferson*
- *fro*[m] [$\mathrm{k^h}$]*anada*
- *fro*[m] [$\mathrm{t^h}$]*acoma*

Borowsky (1986); Avery and Rice (1989); Coleman et al. (2016)

Nasal-place assimilation

- What is a possible long term memory representation?
  - `[Coronal]` is underspecified in nasals.
- What is a possible output of the phonological grammar?
  - Any string that doesn't contain the substrings *np* and *nk*.
- How do we relate these two types of representations?
  - Rule changing /n/ to [m] or [ŋ]
  - Autosegmental feature spreading
  - AGREE(PLACE) $\gg$ IDENT(CORONAL)
  - ...

Nasal-place assimilation

- Production rates (Buckeye Corpus):
  - Pre-labial - 18.5%
  - Pre-velar - 21.8%

- Production rates (Audio British National Corpus):
  - Overall - 20%

- Perception rates (Reverse Inference):
  - Pre-labial - 45%
  - Pre-velar - 35%

Dilley and Pitt (2007); Coleman et al. (2016); Coetzee (2016)

Focus of Today's Talk

The big picture

- How might we formally account for systematic gradient behavior related to phonological knowledge?
- I will argue that mathematical logic provides tools that allows for clarity in expressing the properties of different linguistic systems.
- Furthermore, I will use semirings and weighted logic to distinguish phonological knowledge from phonological usage.

Outline

Philosophical Priors

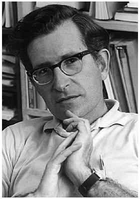
Probabilities should model ignorance, not behavior

"...the reification of probabilities as cognitively represented quantities invites a problematic conception of randomness and probability, in which randomness plays a shallow role in the generation of behavior. This shallow stochasticity is at odds with the role of randomness in physical systems that are the analogues of phonological cognition in the MaxEnt analogy. The states of physical systems are not governed by probabilities. Probabilities are merely statistical descriptions that are useful when we are mostly ignorant about the detailed dynamics of a system and its surroundings."

Tilsen (2023, p. 265)

Grammars should model knowledge, not use

"...it is perhaps worth while to reiterate that a generative grammar is not a model for a speaker or a hearer. It attempts to characterize in the most neutral possible terms the knowledge of the language that provides the basis for actual use of language by a speaker-hearer."



Chomsky (1965, p. 9)

Phonological knowledge is only one of many factors governing behavior

"...linguistic knowledge is only one of the inputs to language production, language comprehension, and other forms of language performance. What accounts for the facts of performance is a conjunct of a theory of linguistic knowledge ('What is the nature of the representation of linguistic knowledge?') and a theory of language performance ('How is this knowledge put to use?')."

Mohanan (1986, p. 183)

Model-Theoretic Phonology

The Basics

- A **model signature** $\mathcal{S}$ is a collection of symbols for the functions, relations, and constants that are used to describe structures; *e.g.,* $\langle \triangleleft, \{R_\sigma \mid \sigma \in \Sigma\} \rangle$.

- An $\mathcal{S}$**-structure** A contains a set called the **domain**, as well as **denotations** for each symbol in $\mathcal{S}$.

- A **logical language** in first-order logic is defined by combining the symbols of first-order logic with a specific model signature $\mathcal{S}$.

Courcelle (1994); Engelfriet and Hoogeboom (2001); Heinz (Forthcoming)

The Basics

- An interpretation of structure A in terms of structure B is a function denoted by a set of n formulas $\{\phi_i, \ldots, \phi_n\}$ where n is equal to the number of functions, relations, and constants in A's model signature, plus a domain formula, copy set, and licensing formula.

- A formula $\phi_P(x) \stackrel{\text{def}}{=} Q(x)$ denotes that domain element x has property P in the output structure only if it has property Q in the input structure.

Courcelle (1994); Engelfriet and Hoogeboom (2001); Heinz (Forthcoming)

Translating a rule into logic

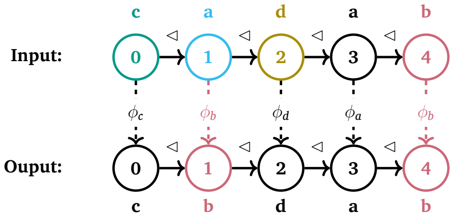
$$a \to b/c\_d$$

$$\phi_a(x) \stackrel{\text{def}}{=} a(x) \wedge \neg\exists y, z[y \lhd x \lhd z \wedge c(y) \wedge d(z)]$$

$$\phi_b(x) \stackrel{\text{def}}{=} b(x) \vee (a(x) \wedge \exists y, z[y \lhd x \lhd z \wedge c(y) \wedge d(z)])$$

$$\phi_c(x) \stackrel{\text{def}}{=} c(x)$$

$$\phi_d(x) \stackrel{\text{def}}{=} d(x)$$



Identifying Properties of Structures

- The model-theoretic approach also provides a way for identifying substructures with logic.
- Typically, these are boolean functions over strings:
  - $f : \Sigma^* \rightarrow \{\texttt{TRUE}, \texttt{FALSE}\}$
- Does structure *A* contain the substring *cad*?
  - $\varphi \stackrel{\text{def}}{=} \exists x[\exists y[y \lhd x \wedge c(y)] \wedge \exists z[x \lhd z \wedge d(z)] \wedge a(x) \wedge \texttt{TRUE}]$

Generalizing the Approach with Weighted Logic

- It is possible to generalize the current approach by changing the type of the co-domain to any *semiring* $\mathbb{S}$.
  - $f : \Sigma^* \to \mathbb{S}$

- A semiring is a set $\mathbb{S}$ that contains..
  - A binary "addition" operator $\oplus$
  - A binary "multiplication" operator $\otimes$
  - A value 0 which is an identity for $\oplus$
  - A value 1 which is an identity for $\otimes$

- Other important properties...

Mohri (1997); Goodman (1999); Roark and Sproat (2007); Gorman and Sproat (2021); Droste and Gastin (2009); Heinz (Forthcoming)

Generalizating the Approach with Weighted Logic

Name	$\mathcal{S}$	$\oplus$	$\otimes$	**0**	**1**
Boolean	$\{\texttt{TRUE}, \texttt{FALSE}\}$	$\vee$	$\wedge$	$\texttt{FALSE}$	$\texttt{TRUE}$
Finite Language	$\texttt{FIN}$	$\cup$	$\cdot$	$\emptyset$	$\{\lambda\}$
Probablity	$\mathbb{R}_{\geq 0}$	$+$	$\times$	0	1
Natural	$\mathbb{N}$	$+$	$\times$	0	1
Viterbi	$[0, 1]$	$\texttt{max}$	$\times$	0	1

- Weighted Logic is MSO Logic where:
  - $s \in S$ is an atomic formula
  - Negation is only allowed in atomic formulas
  - $\phi \wedge \psi$ is interpreted as $\phi \otimes \psi$
  - $\phi \vee \psi$ is interpreted as $\phi \oplus \psi$
  - $\forall x \phi$ is interpreted as $\phi(x_1) \otimes \phi(x_2) \otimes \cdots \otimes \phi(x_n) \quad \forall x \in \mathcal{D}$
  - $\exists x \phi$ is interpreted as $\phi(x_1) \oplus \phi(x_2) \oplus \cdots \oplus \phi(x_n) \quad \forall x \in \mathcal{D}$
  - ...

Finite Language Semiring

- The finite language semiring interprets a single string as a set of strings.
- This provides a way to relate multiple outputs to a single input.
- Automata counterpart is “semi-deterministic”.

- The following equation will map input string *cad* to the output set $\{cad, cbd\}$.
  - $\varphi \overset{\text{def}}{=} \forall x[(a(x) \wedge \{a\}) \vee (b(x) \wedge \{b\}) \vee (c(x) \wedge \{c\}) \vee (d(x) \wedge \{d\}) \vee (\phi_{\text{cad}}(x) \wedge \{b\})]$

Beros and Higuera (2014); Heinz (2020)

Competence and Performance with Weighted Logic

A Basic Sketch

- The basic idea is to split the computation up into two functions; one for phonological **k**nowledge and one for phonological **u**sage.
  - $k : \Sigma^* \to \mathcal{P}(\Sigma^*)$
  - $u : \mathcal{P}(\Sigma^*) \to \mathbb{R}^n_{\geq 0}$
- Competence/Knowledge maps input strings to output sets of strings
- Performance/Usage maps sets of strings to a weight vector
- Output is determined using something like `argmax`.

Phonological Knowledge Function

- Implemented with weighted logic using the finite language semiring.

- Nasal-place assimilation:
  - $\varphi \stackrel{\text{def}}{=} \forall x[(\text{ŋ}(x) \wedge \{\text{ŋ}\}) \vee (n(x) \wedge \{n\}) \vee (m(x) \wedge \{m\}) \vee (n(x) \wedge \texttt{preLabStop}(x) \wedge \{m\}) \vee (n(x) \wedge \texttt{preVelStop}(x) \wedge \{\text{ŋ}\}) \vee (?(x) \wedge \{?\})$

- Extension:
  - (nt,{nt})
  - (np,{np,mp})
  - (nk,{nk,ŋk})
  - (mp,{mp})
  - ...

Phonological Usage Function

- Implemented with weighted logic using the probability semiring.
- This gives a weight to each output of the *k* function – can be turned into a probability distribution by using `softmax`.
  - $\{np, mp\} \rightarrow [4.3, 17.11] \leftrightarrow [0.21, 0.79]$
- Since this is not part of the phonological grammar *proper*, this function can include both phonological and non-phonological information.

Phonological Usage Function

- Factors like *speech rate*, and *experiment type* are argued to influence the rate of assimilation.
  - $\texttt{speechRate} : \{\text{slow,faster,fastest}\} \to \mathbb{R}_{\geq 0}$
  - $\texttt{expType} : \{\text{exp1,exp2}\} \to \mathbb{R}_{\geq 0}$

- It also varies by following place of articulation
  - These are just basic weighted markedness constraints

- $\phi \stackrel{\text{def}}{=} (\texttt{speechRate} \lor \texttt{expType}) \land \forall \boldsymbol{x}[(\boldsymbol{n}(\boldsymbol{x}) \land \texttt{preLabStop}(\boldsymbol{x}) \land \ldots) \lor (\boldsymbol{n}(\boldsymbol{x}) \land \texttt{preVelStop}(\boldsymbol{x}) \land \ldots)]$

Coetzee (2016)

Probabilities?

- What about **The Law of Frequency Matching**?
- Probability matching is a *second order* effect and not the result of probabilities emerging from the grammar.
    - Variation is rarely “free” and is regularly explained by factors both internal and external to the phonological grammar.

Probabilities?

- What about **The Law of Frequency Matching**?
- Probability matching is a *second order* effect and not the result of probabilities emerging from the grammar.
  - If these factors create probability distributions, then one possibility is that language learners identify the factors governing the use of phonological knowledge without directly encoding probabilities in the mind.

Probabilities?

- What about **The Law of Frequency Matching**?
- Probability matching is a *second order* effect and not the result of probabilities emerging from the grammar.
  - Behavior would match the observed probabilities not because speakers have encoded simply the distributions themselves, but because they have identified the factors that lead to a given distribution.

Summary and Conclusion

Summary

- Formal analyses using logic and model theory can account for gradient phenomena.
- The specific analysis requires two functions:
  - A **k**nowledge function which determines what can map to what
  - A **u**sage function which covers which form is used in a specific utterance
- The composition of the two functions suggests they may be viewed as a single function, but only the latter is considered to be affected by extra-grammatical factors and is therefore different in kind from the former.
  - The choice in specific weighted logics also echos the competence/performance distinction.

Conclusion

- Returning to the three philosophical priors:
  1. Probabilities should model ignorance, not behavior.
  2. Grammars should model knowledge, not use.
  3. Phonological knowledge is only one of many factors governing behavior.

- I think that in order for (1) to hold and for (2) to be absent of probabilities, that structured formal models of (3) are needed. Being specific about how phonological knowledge interacts with other factors is the best way to convince others that these are the right philosophical stances to hold.

Nelson and Heinz (in press); Nelson (to appear, 2024); Volenec and Reiss (2017)

THANKYOU!

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