Model Theoretic Phonology and Theory Comparison: Segments, Gestures, and Coupling Graphs

NAPhC 12

Scott Nelson
Stony Brook University

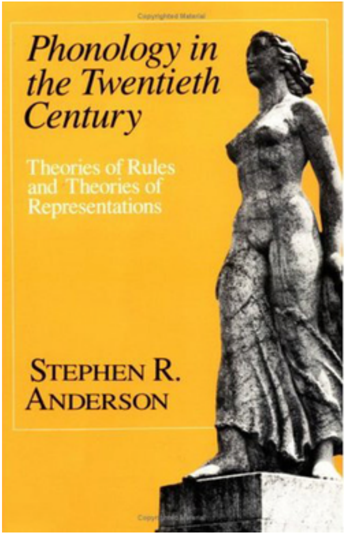
May 13, 2023

Download the slides

- Also available at http://www.scott-nelson.net

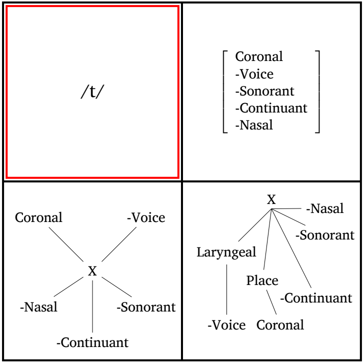
Phonological Representations

- Representations are central to modern phonological theory.
- Many different proposals across phonological domains.



Jakobson et al. (1951); Goldsmith (1976); Clements (1985); Browman and Goldstein (1986); Archangeli (1988); Dresher (2009); Backley (2011); Inkelas and Shih (2016); van der Hulst (2020); *among (many)$^{+}$ others*

Phonological Representations



- Many different proposals throughout the years.
- Focus of this talk will be on segmental representations.

Jakobson et al. (1951); Goldsmith (1976); Clements (1985); Browman and Goldstein (1986); Archangeli (1988); Dresher (2009); Backley (2011); Inkelas and Shih (2016); van der Hulst (2020); *among (many)$^{+}$ others*

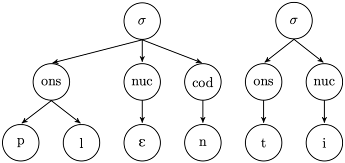
Phonological Representations

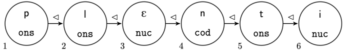
- Recent work has used model theory to compare different proposed representation schemes.
  - Syllable Representations
  - Tonal Geometry
  - Autosegmental/Q Theory
  - Feature Systems

Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

Phonological Representations

- Recent work has used model theory to compare different proposed representation schemes.
  - Syllable Representations
  - Tonal Geometry
  - Autosegmental/Q Theory
  - Feature Systems

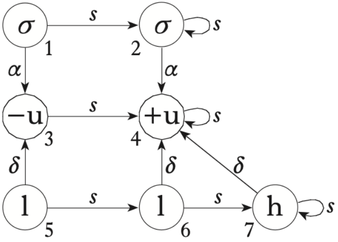


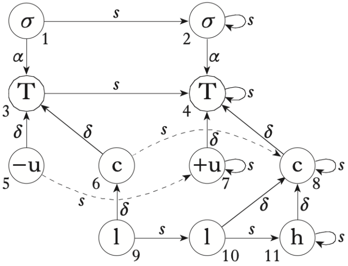


Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

Phonological Representations

- Recent work has used model theory to compare different proposed representation schemes.
  - Syllable Representations
  - Tonal Geometry
  - Autosegmental/Q Theory
  - Feature Systems

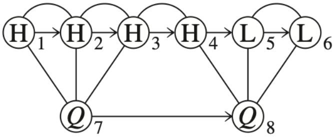


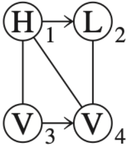


Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

Phonological Representations

- Recent work has used model theory to compare different proposed representation schemes.
  - Syllable Representations
  - Tonal Geometry
  - Autosegmental/Q Theory
  - Feature Systems





Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

Phonological Representations

- Recent work has used model theory to compare different proposed representation schemes.
  - Syllable Representations
  - Tonal Geometry
  - Autosegmental/Q Theory
  - Feature Systems

CPL($\mathcal{M}^v$)	$\mathcal{M}^v_P$	$\mathcal{M}^v_F$	$\mathcal{M}^v_C$
voi	{N,D}	{N,D}	{D}
son	{N}	{N}	{N}
son ∧ voi	{N}	{N}	{}
MISSING	–	{D}, {T}, {D,T}	{T}, {D,T}
EXTRA	–	–	–

CNPL($\mathcal{M}^v$)	$\mathcal{M}^v_P$	$\mathcal{M}^v_F$	$\mathcal{M}^v_C$
voi	{N,D}	{N,D}	{D}
¬voi	{T}	{T}	{N,T}
son	{N}	{N}	{N}
¬son	{D,T}	{D,T}	{D,T}
son ∧ ¬son	{}	{}	{}
son ∧ voi	{N}	{N}	{}
son ∧ ¬voi	{}	{}	{N}
¬son ∧ voi	{D}	{D}	{D}
¬son ∧ ¬voi	{T}	{T}	{T}
voi ∧ ¬voi	{}	{}	{}
MISSING	–	–	–
EXTRA	{D}, {T}, {D,T}	–	{N,T}

Strother-Garcia (2019); Oakden (2020); Jardine et al. (2021); Nelson (2022)

Phonological Representations

- All of the previous work has looked at representations within the larger tradition of generative phonology
  - roughly: input/output mappings described by symbolic changes
- Articulatory Phonology is a theory of phonological representations based in nonlinear dynamics.
  - crucially: no input/output mappings
- **addendum:** many researchers have used gestural representations within generative phonology, but here I am focused on Articulatory Phonology as a theory of phonology that does not have a generative element as it offers a more interesting comparison case.

Browman and Goldstein (1986, 1992); McMahon et al. (1994); Browman and Goldstein (1995); Zsiga (1997); Gafos (2002); Hall (2003); Davidson (2004); Bateman (2007); Nam (2007); Goldstein (2011); Friedman and Visser (2014)

Overview of Talk

- In this talk I will use model theory and logic to show the *bi-interpretability* of segmental strings and coupling graphs which are the lexical representations used in Articulatory Phonology.

Overview of Talk

- **Translation:** Any rule/constraint written about segments can be directly translated into a rule/constraint about coupling graphs in a computationally restricted way.

Model Theoretic Phonology

Model Theoretic Phonology

- What is (finite) model theory?
  - In short: a branch of logic used to study finite structures.
- Why use model theory to study phonology?
  - Allows for flexibility in choices of representation.
  - Gives an idea of the computational complexity required to implement a mapping.
  - Provides a general framework within which specific phonological theories can be studied and compared.

Model Theoretic Phonology - Structure

A model is a tuple $\mathcal{M} = \langle \mathcal{D}, \mathcal{C}, \mathcal{P}, \mathcal{F} \rangle$ where

- $\mathcal{D}$ is a set of domain elements
- $\mathcal{C}$ is a set of constants (not regularly used in phonological models)
- $\mathcal{R}$ is a set of predicates/relations
- $\mathcal{F}$ is a set of functions

Model Theoretic Phonology - Structure

- It's worth pausing for a moment and reflecting on how the different parts of a model theoretic representation relate to a traditional phonological representation.

Model Theoretic Phonology - Structure

- It's worth pausing for a moment and reflecting on how the different parts of a model theoretic representation relate to a traditional phonological representation.
- $\mathcal{M}$ - Whatever representational framework we are working under such as binary features, feature geometry, elements, ...

Model Theoretic Phonology - Structure

- It's worth pausing for a moment and reflecting on how the different parts of a model theoretic representation relate to a traditional phonological representation.
- $\mathcal{D}$ - Identification markers that differentiate the segments, feature bundles, X-slots, and other positional elements in a representation.

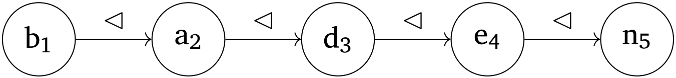
Model Theoretic Phonology - Structure

- It's worth pausing for a moment and reflecting on how the different parts of a model theoretic representation relate to a traditional phonological representation.

- $\mathcal{R}/\mathcal{F}$ - They can be segment labels (IPA symbols) or they can be different phonological or phonetic features. They can also include syllabic, prosodic, and any other necessary information. These also determine ordering relationships between domain elements.

Model Theoretic Phonology - Structure

- A specific collection of domain elements, predicates, and functions is called a **model signature**. One model signatures for strings is:
  - $\langle \mathcal{D}, \mathcal{P}_{\triangleleft}, \mathcal{P}_{\sigma} | \sigma \in \Sigma \rangle$ (successor relation model)
  - Σ is called the **alphabet** and contains the base set of symbols used. (e.g. - $\Sigma = \{a, b\}$ or $\Sigma = \{+voice, +syl, -sonorant, ...\}$)



$$\langle \mathcal{D} = \{1, 2, 3, 4, 5\},$$
$$\lhd = \{(1, 2), (2, 3), (3, 4), (4, 5)\},$$
$$\text{b} = \{1\}, \text{a} = \{2\}, \text{d} = \{3\}, \text{e} = \{4\}, \text{n} = \{5\}\rangle$$

Model Theoretic Phonology - Translations

- Given a specific model, it is possible to define a **logical translation** from one structure into another structure.
- Translating one structure into another structure is exactly what phonology does.
- Therefore, these logical translations can be thought of as declarative statements that describe phonological maps.

Courcelle (1994); Engelfriet and Hoogeboom (2001); Heinz (202x)

Model Theoretic Phonology - Translations

- Translation is done by defining output properties of a structure in terms of input properties.
  - Formulae such as $\phi_P(x) = Q(x)$ are interpreted as "domain element x has property P in the output structure if it has property Q in the input structure".
  - Every predicate/function gets an output formula
  - Additionally, one must specify how many copies of the input domain are needed (>1 if epenthesis or reduplication; else $=1$) and which copies are licensed in the output (TRUE unless deletion occurs).

Courcelle (1994); Engelfriet and Hoogeboom (2001); Heinz (202x)

Model Theoretic Phonology - Translations

- Given a rule $a \rightarrow b/c\_d$
- $\phi_{\mathrm{b}}(x) \stackrel{\mathrm{def}}{=} \mathrm{b}(x) \vee [\mathrm{a}(x) \wedge \mathrm{c}(p(x)) \wedge \mathrm{d}(s(x))]$
  - $\phi_{\mathrm{b}}(x) \stackrel{\mathrm{def}}{=}$ ... "make domain element x a b on the output if..."
  - $\mathrm{b}(x) \vee$ ... "x is a b on the input or..."
  - $[\mathrm{a}(x) \wedge \mathrm{c}(p(x)) \wedge \mathrm{d}(s(x))]$ ... "x is an a on the input and preceded by a c and followed by a d.

$s(x)$ and $p(x)$ are successor and predecessor **functions** with types $\mathcal{D} \rightarrow \mathcal{D}$.

Model Theoretic Phonology - Translations

- Some actual linguistic data...aka the “Heather slide”.
- German final devoicing.
  1. /bad+en/ $\rightarrow$ [baden] ‘to bathe’
  2. /bad/ $\rightarrow$ [bat] ‘bath’
  3. bat+en/ $\rightarrow$ [baten] ‘asked’
  4. /bat/ $\rightarrow$ [bat] ‘ask’

Dinnsen and Garcia-Zamor (1971)

Model Theoretic Phonology - Translations

- The following set of formulae describe the final devoicing mapping.

$$\texttt{final}(x) \stackrel{\text{def}}{=} s(x) = x$$

$$\phi_{\texttt{domain}} \stackrel{\text{def}}{=} \texttt{TRUE}$$

$$C \stackrel{\text{def}}{=} \{1\}$$

$$\phi_{\texttt{license}} \stackrel{\text{def}}{=} \texttt{TRUE}$$

$$\phi_{\text{a}}(x) \stackrel{\text{def}}{=} \text{a}(x)$$

$$\phi_{\text{e}}(x) \stackrel{\text{def}}{=} \text{e}(x)$$

$$\phi_{\text{n}}(x) \stackrel{\text{def}}{=} \text{n}(x)$$

$$\phi_{\text{b}}(x) \stackrel{\text{def}}{=} \text{b}(x) \wedge \neg\texttt{final}(x)$$

$$\phi_{\text{p}}(x) \stackrel{\text{def}}{=} \text{p}(x) \vee (\text{b}(x) \wedge \texttt{final}(x))$$

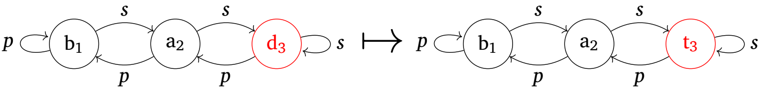
$$\phi_{\text{d}}(x) \stackrel{\text{def}}{=} \text{d}(x) \wedge \neg\texttt{final}(x)$$

$$\phi_{\text{t}}(x) \stackrel{\text{def}}{=} \text{t}(x) \vee (\text{d}(x) \wedge \texttt{final}(x))$$

$$\phi_{s}(x) \stackrel{\text{def}}{=} s(x)$$

$$\phi_{p}(x) \stackrel{\text{def}}{=} p(x)$$

Model Theoretic Phonology - Translations

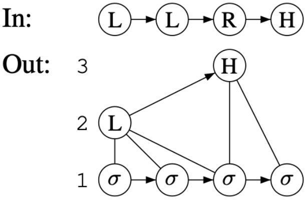


- Domain element 3 gets changed from **d** to **t** because it satisfies this formula.
- $\phi_{\mathrm{t}}(x) \stackrel{\text{def}}{=} \mathrm{t}(x) \vee (\mathrm{d}(x) \wedge \mathtt{final}(x))$

Model Theoretic Phonology - Translations

- There is no limitation on what type of structures one can translate between.
- In other words, the same technique can be used to translate between different representation schemes.
- This allows for direct comparisons of equality and expressivity.

Model Theoretic Phonology - Translations



$$\varphi^{1,1}_{\triangleleft'}(x,y) \stackrel{\text{def}}{=} x \triangleleft y$$
$$\varphi^{2,2}_{\triangleleft'}(x,y) \stackrel{\text{def}}{=} x \prec y \wedge \mathtt{spanfirst}(x) \wedge \mathtt{spanfirst}(y) \wedge (\forall z)[x \prec z \prec y \rightarrow \mathtt{span}(x,z)]$$
$$\varphi^{2,3}_{\triangleleft'}(x,y) \stackrel{\text{def}}{=} \big(\mathtt{spanfirst}(x) \wedge (\mathrm{F}(y) \vee \mathrm{R}(y)) \wedge x = y\big) \vee \big(\mathtt{spanfirst}(x) \wedge (\mathrm{F}(x) \vee \mathrm{R}(x)) \wedge \mathtt{span}(x,y)\big)$$
$$\varphi^{3,2}_{\triangleleft'}(x,y) \stackrel{\text{def}}{=} (\mathrm{F}(x) \vee \mathrm{R}(x)) \wedge \mathtt{spanfirst}(y) \wedge \mathtt{span}(x,y)$$
$$\varphi^{3,3}_{\triangleleft'}(x,y) \stackrel{\text{def}}{=} (\mathrm{F}(x) \vee \mathrm{R}(x)) \wedge (\mathrm{F}(y) \vee \mathrm{R}(y)) \wedge \mathtt{span}(x,y)$$

- “The second, positive, result is that [Autosegmental Representation]s are [First Order]-definable from strings, showing that they do not significantly increase the expressive power of phonotactic grammars. It is thus also likely that they do not significantly increase the expressive power of string mappings, although the logical study of phonological transformations is still ongoing [SN 2023 update: they don’t].”

Jardine (2017); Lambert (2022)

Articulatory Phonology

Gestural Representations

- A **gesture** is "a characteristic pattern of movement of an articulator (or of an articulatory subsystem) through space, over time" (p. 237)
- "We take, then, as a first hypothesis that gestures can be characterised in terms of a dynamical system and its associated motion variables and parameter values [e.g. constriction location/degree]..." (p. 240).
- $m\ddot{x} + k(x - x_0) = 0$

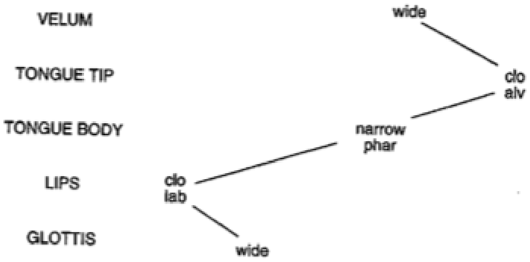
Browman and Goldstein (1986, 1995)

Gestural Representations

- "...a **constellation** of gestures is a set of gestures that are coordinated with one another by means of phasing..." (p. 185).
- "Each gesture is assumed to be active for a fixed proportion of its virtual cycle...The linguistic gestural model uses this proportion, along with the stiffness of each gesture and the phase relations among the gestures, to calculate a *gestural score* that specifies the temporal activation intervals for each each gesture in an utterance" (p. 187).
- "The parameter value specifications and activation intervals from the gestural score are input to the task-dynamical model..., which calculates the time-varying response of the tract variable and component articulators to the imposition of the dynamical regimes defined by the gestural score" (p. 188).

Browman and Goldstein (1986, 1995)

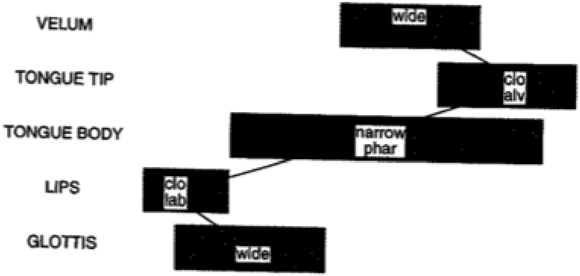
Gestural Representations



- Representations of [pɑn]
  - Constellation
  - Gestural Score
  - Trajectories

Browman and Goldstein (1986, 1995)

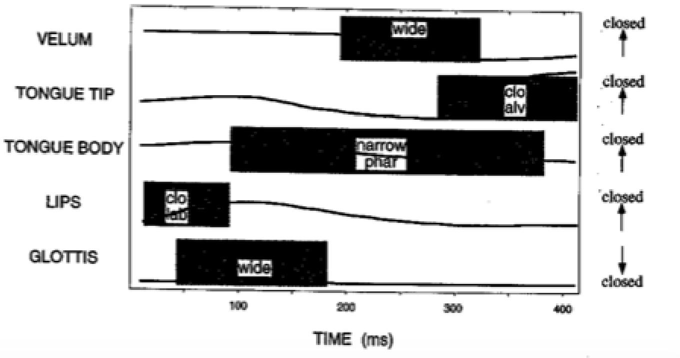
Gestural Representations



- Representations of [pɑn]
  - Constellation
  - Gestural Score
  - Trajectories

Browman and Goldstein (1986, 1995)

Gestural Representations



- Representations of [pɑn]
  - Constellation
  - Gestural Score
  - Trajectories

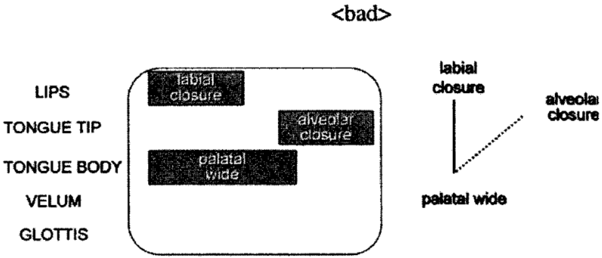
Browman and Goldstein (1986, 1995)

Coupling Graphs as Lexical Representations

"In previous work, the gestural scores were constructed using rules that specified the relative phase of pairs of gestures...In this model, planning oscillators associated with the set of gestures in a given utterance are coupled in a pairwise, bidirectional manner specified in a coupling graph (or structure) that is part of **the lexical specification of a word"** *(p. 38).*

Nam (2007)

Coupling Graphs as Lexical Representations



- Solid line is in-phase (0 degrees; simultaneous)
- Dashed line is anti-phase (180 degrees; opposite)

Nam (2007)

Model Theoretic Representations

Coupling Graph Models

Relation	Label
$\diamond$	In-phase
$\lhd_{180}$	Anti-phase
$\lhd_{60}$	Abutting
$\lhd_{30}$	Eccentric

- 4 binary relations based on common phase relations in Articulatory Phonology

Coupling Graph Models

Relation	Label	Relation	Label
`LIPS`	Labial Articulator	`rel`	Constriction Degree: release
`TT`	Tongue Tip Articulator	`pro`	Constriction Location: protruded
`TB`	Tongue Body Articulator	`dent`	Constriction Location: dental
`VEL`	Velum Articulator	`alv`	Constriction Location: alveolar
`GLO`	Glottis Articulator	`palv`	Constriction Location: postalveolar
`clo`	Constriction Degree: closed	`pal`	Constriction Location: palatal
`crit`	Constriction Degree: critical	`vel`	Constriction Location: velar
`nar`	Constriction Degree: narrow	`uvul`	Constriction Location: uvular
`V`	Constriction Degree: vowel	`phar`	Constriction Location: pharyngeal
`wide`	Constriction Degree: wide		

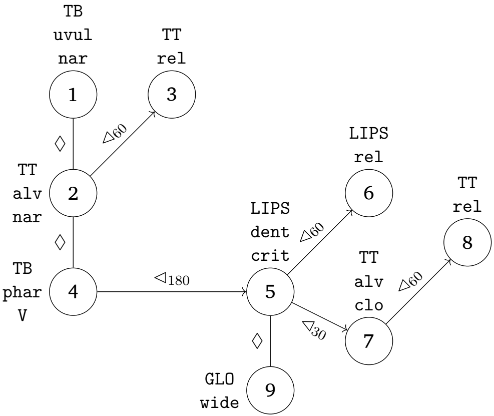
- Unary labeling relations.

Coupling Graph Model: [læft]

$$\mathcal{D} := \{1,2,3,4,5,6,7,8,9\}$$
$$\diamond := \{(1,2),(2,4),(5,9)\}$$
$$\lhd_{180} := \{(4,5)\}$$
$$\lhd_{60} := \{(2,3),(5,6),(7,8)\}$$
$$\lhd_{30} := \{(5,7)\}$$
$$\texttt{LIPS} := \{5,6\}$$
$$\texttt{TT} := \{2,3,7,8\}$$
$$\texttt{TB} := \{1,4\}$$
$$\texttt{GLO} := \{9\}$$

$$\texttt{dent} := \{5\}$$
$$\texttt{alv} := \{2,7\}$$
$$\texttt{uvul} := \{1\}$$
$$\texttt{phar} := \{4\}$$
$$\texttt{clo} := \{7\}$$
$$\texttt{crit} := \{5\}$$
$$\texttt{nar} := \{1,2\}$$
$$\texttt{wide} := \{9\}$$
$$\texttt{rel} := \{3,6,8\}$$
$$\texttt{V} := \{4\}$$

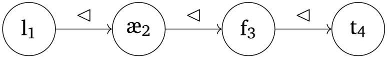
Coupling Graph Model: [læft]



String Model: [læft]

Relation	Label
$\lhd$	Successor
$\sigma (\forall \sigma \in \Sigma)$	Segment

$$\begin{aligned}
\langle \mathcal{D} &:= \{1, 2, 3, 4\} \\
\lhd &:= \{(1,2), (2,3), (3,4)\} \\
\text{æ} &:= \{2\} \\
\text{f} &:= \{3\} \\
\text{l} &:= \{1\} \\
\text{t} &:= \{4\} \\
\sigma &:= \{\}; \sigma \in \Sigma \setminus \{\text{æ,f,l,t}\}\rangle
\end{aligned}$$

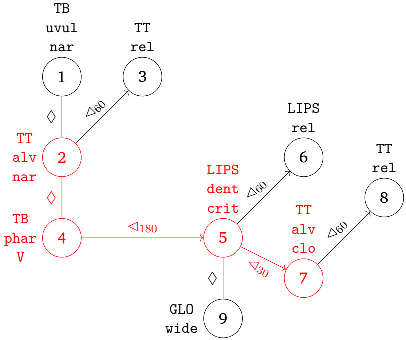


Translations

Translating between Structures

- Here we'll define two translations:
  - Coupling graph to string: Γ^{sg}
  - String to coupling graph: Γ^{gs}

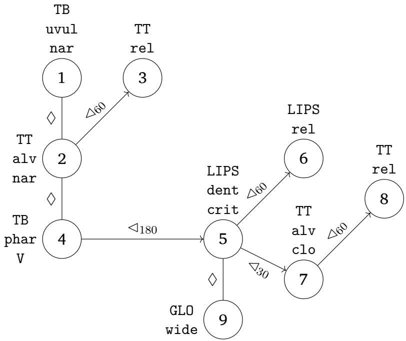
Identifying the "spine"



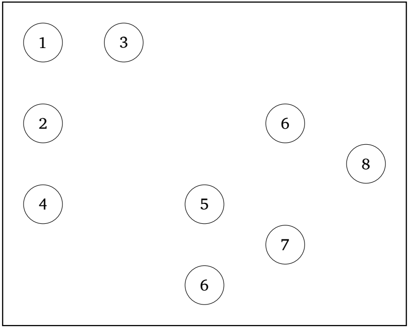
- We can identify the *spine* of a coupling graph by looking at the subgraph that does not include:
  - Secondary articulations
  - Release Gestures
  - Glottal Gestures
  - Velum Gestures

Translating from coupling graph to string

Input



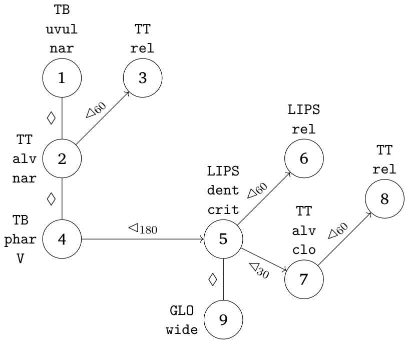
Workspace



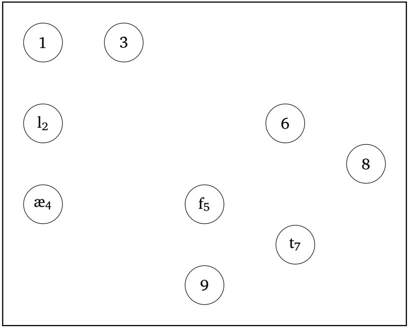
- $C := \{1\}$

Translating from coupling graph to string

Input



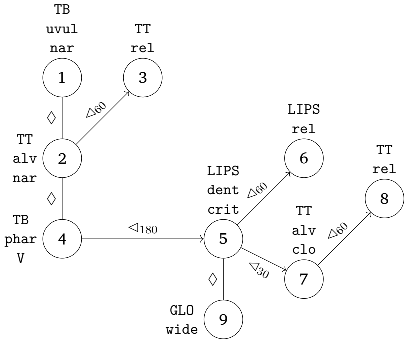
Workspace



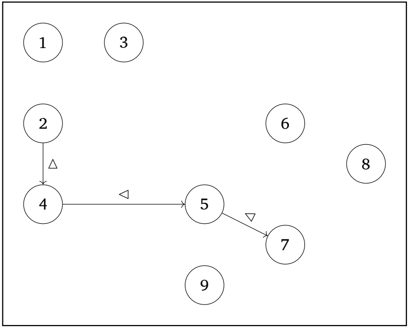
- $\varphi_{\mathrm{l}}(x) := \mathtt{TT}(x) \wedge \mathtt{alv}(x) \wedge \mathtt{nar}(x) \wedge \exists y[x \diamond y \wedge \mathtt{voc}(y) \wedge \mathtt{TB}(y) \wedge \mathtt{uvul}(y)]$
- $\varphi_{\mathrm{æ}}(x) := \mathtt{TB}(x) \wedge \mathtt{phar}(x) \wedge \mathtt{V}(x)$
- $\varphi_{\mathrm{f}}(x) := \mathtt{LIPS}(x) \wedge \mathtt{dent}(x) \wedge \mathtt{crit}(x) \wedge \exists y[x \diamond y \wedge \mathtt{GLO}(y) \wedge \mathtt{wide}(y)]$
- $\varphi_{\mathrm{t}}(x) := \mathtt{TT}(x) \wedge \mathtt{alv}(x) \wedge \mathtt{clo}(x) \wedge ((\exists yz[y \lhd_{30} x \Rightarrow (y \diamond z \wedge \mathtt{GLO}(z))]) \vee (\exists y[x \diamond y \wedge \mathtt{GLO}(y) \wedge \mathtt{wide}|(y)]))$

Translating from coupling graph to string

Input



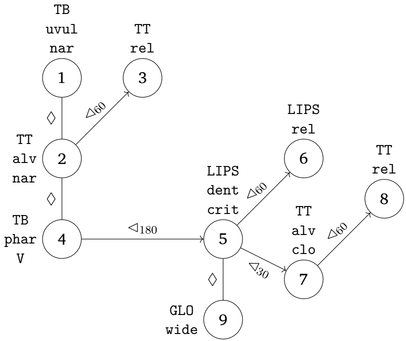
Workspace



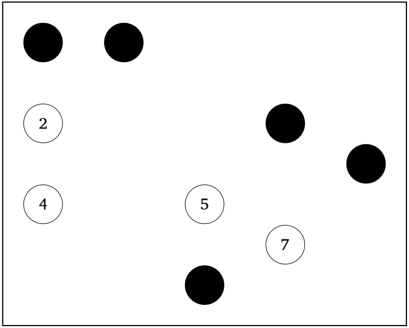
- Onset Cs are in phase with V and anti-phase with preceding C.
- First coda C is anti-phase with V; all other Cs eccentric with preceding C.
- $\varphi_{\triangleleft}(x, y) := (x \triangleleft_{180} y) \vee (x \triangleleft_{30} y) \vee (x \Diamond y \wedge \mathtt{V}(y) \wedge \neg\exists z[x \triangleleft_{180} z])$

Translating from coupling graph to string

Input



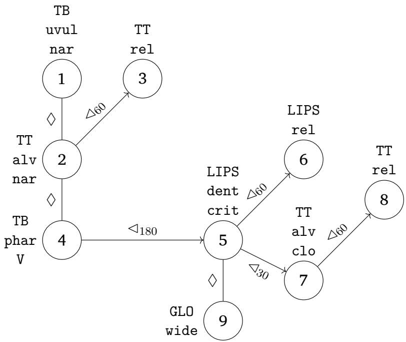
Workspace



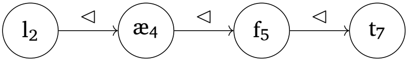
- "spine" identification.
- $\varphi_{license}(x) := \neg\texttt{rel}(x) \wedge \neg\texttt{GLO}(x) \wedge ((\texttt{TB}(x) \wedge \neg\texttt{V}(x)) \Rightarrow \neg\exists y[\texttt{TT}(y) \wedge x \Diamond y])$

Translating from coupling graph to string

Input



Output

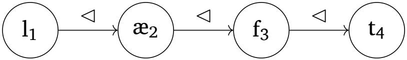


Expansion

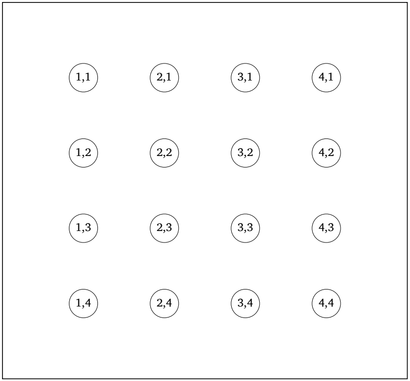
- Going from coupling graph to string removes information.
- What happens when we have to expand the representation and add more information by going from a string to a coupling graph?
- Spoiler: no real problems arise

Translating from string to coupling graph

Input



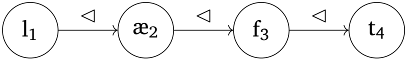
Workspace



- $C := \{1, 2, 3, 4\}$
- Unique copy sets for primary gesture, release gesture, secondary gestures, glottal/nasal gesture

Translating from string to coupling graph

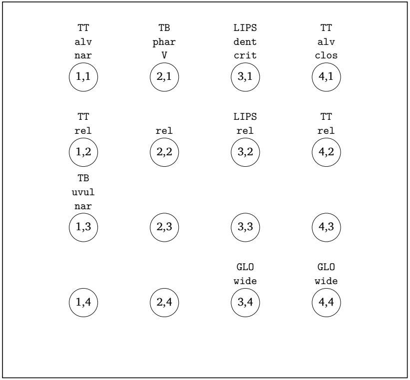
Input



$$\begin{aligned}
\varphi^1_{\texttt{LIPS}}(x) &:= \mathrm{f}(x) \\
\varphi^2_{\texttt{LIPS}}(x) &:= \varphi^1_{\texttt{LIPS}}(x) \\
\varphi^1_{\texttt{TT}}(x) &:= \mathrm{t}(x) \lor \mathrm{l}(x) \\
\varphi^2_{\texttt{TT}}(x) &:= \varphi^1_{\texttt{TT}}(x) \\
\varphi^1_{\texttt{TB}}(x) &:= \text{æ}(x) \\
\varphi^3_{\texttt{TB}}(x) &:= \mathrm{l}(x) \\
\varphi^4_{\texttt{GLO}}(x) &:= \mathrm{t}(x) \lor \mathrm{f}(x) \\
\varphi^1_{\texttt{dent}} &:= \mathrm{f}(x) \\
\varphi^1_{\texttt{alv}} &:= \mathrm{t}(x)
\end{aligned}$$

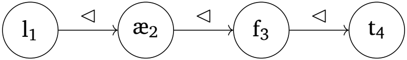
$$\begin{aligned}
\varphi^1_{\texttt{phar}} &:= \text{æ}(x) \\
\varphi^3_{\texttt{uvul}} &:= \mathrm{l}(x) \\
\varphi^1_{\texttt{clo}} &:= \mathrm{t}(x) \\
\varphi^1_{\texttt{crit}} &:= \mathrm{f}(x) \\
\varphi^1_{\texttt{V}} &:= \text{æ}(x) \\
\varphi^1_{\texttt{nar}} &:= \mathrm{l}(x) \\
\varphi^3_{\texttt{nar}} &:= \mathrm{l}(x) \\
\varphi^4_{\texttt{wide}} &:= \mathrm{t}(x) \lor \mathrm{f}(x)
\end{aligned}$$

Workspace



Translating from string to coupling graph

Input



$$\varphi^{1,1}_{\diamond}(x,y) := x \lhd y \wedge \text{æ}(y) \wedge \neg\text{æ}(x)$$

$$\varphi^{1,3}_{\diamond}(x,y) := (x = y) \wedge \text{l}(x)$$

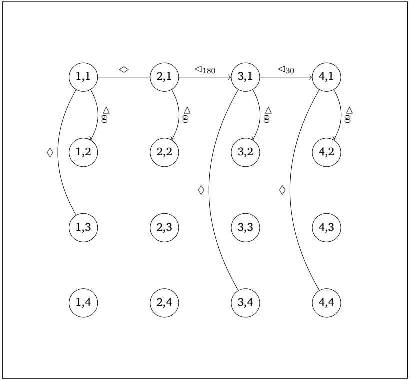
$$\varphi^{1,4}_{\diamond}(x,y) := (x = y) \wedge \text{t}(x) \vee \text{f}(x)$$

$$\varphi^{1,1}_{\lhd_{180}}(x,y) := x \lhd y \wedge \text{æ}(x) \wedge \neg\text{æ}(y)$$

$$\varphi^{1,2}_{\lhd_{60}}(x,y) := (x = y)$$

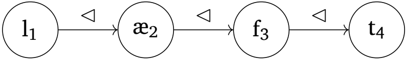
$$\varphi^{1,1}_{\lhd_{30}}(x,y) := \neg\text{æ}(x) \wedge \neg\text{æ}(y) \wedge \exists z[z \lhd x \wedge \text{æ}(z)]$$

Workspace



Translating from string to coupling graph

Input



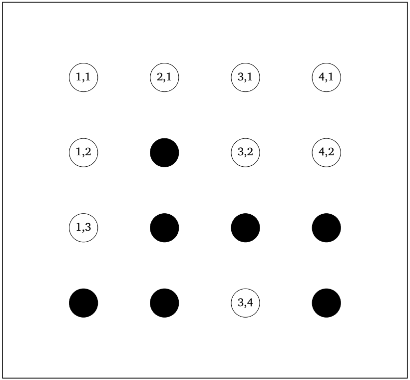
$$\varphi^{1}_{license}(x) := \texttt{True}$$

$$\varphi^{2}_{license}(x) := \mathrm{f}(x) \lor \mathrm{t}(x) \lor \mathrm{l}(x)$$

$$\varphi^{3}_{license}(x) := \mathrm{l}(x)$$

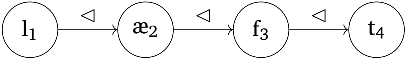
$$\varphi^{4}_{license}(x) := \mathrm{t}(x) \lor \mathrm{f}(x) \land \neg\exists y[y \lhd x \land \mathrm{f}(y) \lor \mathrm{t}(y)]$$

Workspace

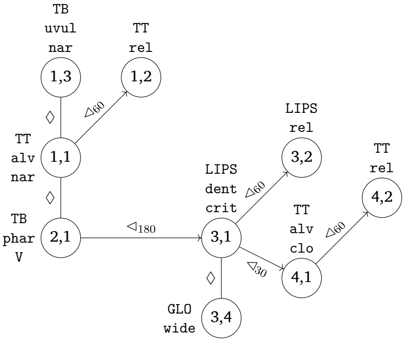


Translating from string to coupling graph

Input



Output



Bi-Interpretability

Definition:

We note that an interpretation $K : U \to V$ gives us a construction of an internal model $\widetilde{K}(\mathcal{M})$ of U from a model M of V. We find that U and V are bi-interpretable iff, there are interpretations $K : U \to V$ and $M : V \to U$ and formulas F and G such that, for all models $\mathcal{M}$ of V, the formula F defines an isomorphism between $\mathcal{M}$ and $\widetilde{M}\widetilde{K}(\mathcal{M})$, and, for all models $\mathcal{N}$ of U, the formula G defines an isomorphism between $\mathcal{N}$ and $\widetilde{K}\widetilde{M}(\mathcal{N})$.

Friedman and Visser (2014); Oakden (2020)

Bi-Interpretability

$\mathcal{M}^s$	string model of *laughed*
$\mathcal{M}^g$	coupling graph model of *laughed*
Γ^{sg}	string to coupling graph transduction
Γ^{gs}	coupling graph to string transduction

- $\mathcal{M}^s \equiv \Gamma^{gs}(\Gamma^{sg}(\mathcal{M}^s))$
- $\mathcal{M}^g \equiv \Gamma^{sg}(\Gamma^{gs}(\mathcal{M}^g))$
- This indicates the string and coupling graph models are **bi-interpretable**

Friedman and Visser (2014); Oakden (2020)

Conclusion

It's symbols all the way down

Thus we are referring to the same set of dynamically specified gestures, but this time using symbols which serve as indices to entire dynamical systems. **These symbolic descriptions highlight those aspects of the gestural structures that are relevant for contrast among lexical items**" *(p. 241)*.

Browman and Goldstein (1986)

General Takeaways

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.

General Takeaways

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.
- Within this sphere we get bi-interpretability with string representations.

General Takeaways

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.
- Within this sphere we get bi-interpretability with string representations.
- The string to coupling graph translation is unsurprising:

General Takeaways

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.
- Within this sphere we get bi-interpretability with string representations.
- The string to coupling graph translation is unsurprising:

> Note: while the other models represented by boxes in Figure 1 (**Coupled oscillator model of inter-gestural coordination** and **Task dynamic model of inter-articulator coordination)** are meant to be part of a model of the human speech production process, the method used for automatic generation of coupling graphs is a heuristic that is not meant to model how a speaker would go about construction a coupling graph for an arbitrary form. Coupling graphs could simply be stored by speakers in the lexicon. The automatic computation has two major benefits: (1) It represents in compact form generalizations about the set of coupling graphs that speakers use (in English, at least) and their relation to more conventional phonological representations (segments, features, syllable structure). (2) It allows the later stages of the model to be tested, by allowing automatic generation of a variety input files.

Saltzman (2001)

General Takeaways

- The assumptions about coupling graph structure make it so only certain types of gestural representations are considered.
- Within this sphere we get bi-interpretability with string representations.
- The string to coupling graph translation is unsurprising:

> Note: while the other models represented by boxes in Figure 1 (**Coupled oscillator model of inter-gestural coordination** and **Task dynamic model of inter-articulator coordination)** are meant to be part of a model of the human speech production process, the method used for automatic generation of coupling graphs is a heuristic that is not meant to model how a speaker would go about construction a coupling graph for an arbitrary form. Coupling graphs could simply be stored by speakers in the lexicon. The automatic computation has two major benefits: (1) It represents in compact form generalizations about the set of coupling graphs that speakers use (in English, at least) and their relation to more conventional phonological representations (segments, features, syllable structure). (2) It allows the later stages of the model to be tested, by allowing automatic generation of a variety input files.

- But the coupling graph to string translation is novel (Jason Shaw, p.c.).

Saltzman (2001)

Further Directions

- Implement full translation for English

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses
  - E.g. - constraints over segments are easy to write but may be cumbersome for a coupling graph structure

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses
  - E.g. - constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses
  - E.g. - constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
  - Sets of coupling graphs map to strings

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses
  - E.g. - constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
  - Sets of coupling graphs map to strings
  - Strings map to sets of coupling graphs

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses
  - E.g. - constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
  - Sets of coupling graphs map to strings
  - Strings map to sets of coupling graphs
  - Could explain within- and between-language variation

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses
  - E.g. - constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
  - Sets of coupling graphs map to strings
  - Strings map to sets of coupling graphs
  - Could explain within- and between-language variation
- Is string vs. coupling graph the right comparison?

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses
  - E.g. - constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
  - Sets of coupling graphs map to strings
  - Strings map to sets of coupling graphs
  - Could explain within- and between-language variation
- Is string vs. coupling graph the right comparison?
  - If the comparison is really about input-ouput mappings vs. non input-output mappings then maybe a two-level string correspondence graph is a better way to compare the two theories?

Further Directions

- Implement full translation for English
  - Provides a bridge for analyses done in AP vs. string-based analyses
  - E.g. - constraints over segments are easy to write but may be cumbersome for a coupling graph structure
- Explore the ways in which the continuous parameters underlying the gesture dynamics interact with the translations.
  - Sets of coupling graphs map to strings
  - Strings map to sets of coupling graphs
  - Could explain within- and between-language variation
- Is string vs. coupling graph the right comparison?
  - If the comparison is really about input-ouput mappings vs. non input-output mappings then maybe a two-level string correspondence graph is a better way to compare the two theories?
- Can we change the representations slightly to write a QF transduction?

Final Thoughts

- Model theory provides a meta-language to do cross-theory comparison in phonology.

Final Thoughts

- Model theory provides a meta-language to do cross-theory comparison in phonology.
- Here, I showed that coupling graphs don't encode more information than strings, they just encode it differently.

Final Thoughts

- Model theory provides a meta-language to do cross-theory comparison in phonology.
- Here, I showed that coupling graphs don't encode more information than strings, they just encode it differently.
- Furthermore, the computational power needed to translate between the two is restricted and within the domain needed to express phonological generalizations and phonotactic restrictions.

Final Thoughts

- This highlights that it is not the representations that are different between the two theories, but rather how the representations are interpreted.
  - AP coupling graphs already contain all the necessary information for phonetic implementation.
  - Strings must be further interpreted somehow (but we've seen that's not too difficult to do).

Final Thoughts

- This highlights that it is not the representations that are different between the two theories, but rather how the representations are interpreted.
  - AP coupling graphs already contain all the necessary information for phonetic implementation.
  - Strings must be further interpreted somehow (but we've seen that's not too difficult to do).

Final Thoughts

- This highlights that it is not the representations that are different between the two theories, but rather how the representations are interpreted.
  - AP coupling graphs already contain all the necessary information for phonetic implementation.
  - Strings must be further interpreted somehow (but we've seen that's not too difficult to do).

Thank You!

Bibliography I

Archangeli, D. (1988). Aspects of underspecification theory. Phonology, 5(2):183–207.

Backley, P. (2011). Introduction to element theory. Edinburgh University Press.

Bateman, N. (2007). A crosslinguistic investigation of palatalization. University of California, San Diego.

Browman, C. P. and Goldstein, L. (1992). Articulatory phonology: An overview. Phonetica, 49(3-4):155–180.

Browman, C. P. and Goldstein, L. (1995). Dynamics and articulatory phonology. Mind as motion, 175:193.

Browman, C. P. and Goldstein, L. M. (1986). Towards an articulatory phonology. Phonology, 3:219–252.

Clements, G. N. (1985). The geometry of phonological features. Phonology, 2(1):225–252.

Courcelle, B. (1994). Monadic second-order definable graph transductions: a survey. Theoretical Computer Science, 126(1):53–75.

Davidson, L. (2004). The atoms of phonological representation: Gestures, coordination and perceptual features in consonant cluster phonotactics. The Johns Hopkins University.

Dinnsen, D. and Garcia-Zamor, M. (1971). The three degrees of vowel length in German. Research on Language & Social Interaction, 4(1):111–126.

Dresher, B. E. (2009). The contrastive hierarchy in phonology. Number 121. Cambridge University Press.

Engelfriet, J. and Hoogeboom, H. J. (2001). MSO definable string transductions and two-way finite-state transducers. ACM Transactions on Computational Logic (TOCL), 2(2):216–254.

Friedman, H. M. and Visser, A. (2014). When bi-interpretability implies synonymy. Logic Group Preprint Series, 320:1–19.

Gafos, A. I. (2002). A grammar of gestural coordination. Natural language & linguistic theory, 20(2):269–337.

Goldsmith, J. A. (1976). Autosegmental phonology. PhD thesis, Massachusetts Institute of Technology.

Goldstein, L. (2011). Back to the past tense in English. In Representing language: Essays in honor of Judith Aissen, pages 69–88.

Hall, N. E. (2003). Gestures and segments: Vowel intrusion as overlap. University of Massachusetts Amherst.

Heinz, J. (202x). Doing computational phonology. Unpublished Manuscript.

Inkelas, S. and Shih, S. S. (2016). Re-representing phonology: consequences of q theory. In Proceedings of NELS, volume 46, pages 161–174.

Bibliography II

Jakobson, R., Fant, C. G., and Halle, M. (1951). Preliminaries to speech analysis: The distinctive features and their correlates.

Jardine, A. (2017). On the logical complexity of autosegmental representations. In Proceedings of the 15th Meeting on the Mathematics of Language, pages 22–35.

Jardine, A., Danis, N., and Iacoponi, L. (2021). A formal investigation of q-theory in comparison to autosegmental representations. Linguistic Inquiry, 52(2):333–358.

Lambert, D. (2022). Unifying Classification Schemes for Languages and Processes With Attention to Locality and Relativizations Thereof. PhD thesis.

McMahon, A., Foulkes, P., and Tollfree, L. (1994). Gestural representation and lexical phonology. Phonology, 11(2):277–316.

Nam, H. (2007). A gestural coupling model of syllable structure. PhD thesis, Yale University.

Nelson, S. (2022). A model theoretic perspective on phonological feature systems. Proceedings of the Society for Computation in Linguistics, 5(1).

Oakden, C. (2020). Notational equivalence in tonal geometry. Phonology, 37(2):257–296.

Saltzman, E. (2001). Tada (task dynamics application) manual.

Strother-Garcia, K. (2019). Using model theory in phonology: a novel characterization of syllable structure and syllabification. PhD thesis, University of Delaware.

van der Hulst, H. (2020). Principles of Radical CV Phonology: A theory of segmental and syllabic structure. Edinburgh University Press.

Zsiga, E. C. (1997). Features, gestures, and igbo vowels: An approach to the phonology-phonetics interface. Language, pages 227–274.