



Computing Long-Distance Phenomena with Boolean Monadic Recursive Schemes

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What are BMRS?

- BMRS** ^[ˈbiːmɹɔːz] = **B**oolean **M**onadic **R**ecursive **S**chemes (Bhaskar , 2020, 2023)
- A **BMRS program** ρ defines a string-to-string function via a collection of recursive functions $\{\mathbf{f}_1, \dots, \mathbf{f}_n\}$, a subset of which are labeled ϕ_f and determine whether an output string carries a given feature property f .
- These functions use **IF...THEN...ELSE** syntax, take singular domain elements as input (hence *monadic*), and have an output type of **Boolean** $\in \{\top, \perp\}$.

How to BMRS

Inductive definition of BMRS functions (adapted from Chandlee & Jardine, 2021, 492)

- $\top$ (= true) and $\perp$ (= false) are expressions
- any predicate $P(t)$ is an expression
 - P may be a feature ($[+\mathbf{f}] = \top$, $[-\mathbf{f}] = \perp$), or a boundary ($\bowtie$ = initial, $\bowtie$ = final), etc.
 - t is the string position under evaluation (x), its predecessor ($p(x)$), or its successor ($s(x)$)
- if E_1 , E_2 , and E_3 are expressions, then so is ‘**IF** E_1 **THEN** E_2 **ELSE** E_3 ’
- nothing else is an expression

Example BMRS functions

Non-recursive progressive [round] assimilation

- $\phi_{\text{round}}(x) := \text{IF } \text{init}(x) \text{ THEN round}(x) \text{ ELSE round}(p(x))$

- The output value of [round] of each segment x is determined as follows: if the input value of **init** of x is $\top$, then the output value of [round] is faithful to its underlying value. Otherwise, it is determined based on the **input** value of [round] of the preceding segment.

Recursive progressive [round] spreading

- $\phi_{\text{round}}(x) := \text{IF } \text{init}(x) \text{ THEN round}(x) \text{ ELSE } \phi_{\text{round}}(p(x))$

- The output value of [round] of each segment x is determined as follows: if the input value of **init** of x is $\top$, then the output value of [round] is faithful to its underlying value. Otherwise, it is determined based on the **output** value of [round] of the preceding segment.

Example BMRS evaluations

- Both types of patterns are found in dialects of Crimean Tatar. (McCollum & Kavitskaya, 2022)

Non-recursive assimilation

E.g. Central Crimean Tatar

/tuz-luy-ɯ/ $\rightarrow$ [tuz-luy-ɯ] (salt-NOUN-POSS.3)

Input	R	U	U
x	1	2	3
init(x)	$\top$	$\perp$	$\perp$
round(x)	$\top$	$\perp$	$\perp$
$\phi_{\text{round}}(x)$	$\top$	$\top$	$\perp$
Output	R	R	U

Recursive spreading

E.g. Southern Crimean Tatar

/tuz-luy-ɯ/ $\rightarrow$ [tuz-luy-ɯ] (salt-NOUN-POSS.3)

Input	R	U	U
x	1	2	3
init(x)	$\top$	$\perp$	$\perp$
round(x)	$\top$	$\perp$	$\perp$
$\phi_{\text{round}}(x)$	$\top$	$\top$	$\top$
Output	R	R	R

Main Takeaway

Chandlee & Jardine (2021, p. 514) point to long-distance phenomena as an open area of inquiry within the BMRS framework. We show that there are multiple ways to intentionally characterize these phenomena within the formalism but hypothesize that there may be a distinction in how certain types of long distance phenomena are computed.

Search & Copy/Change with BMRS

- Search-based theories are *target-centric* such that an INITIATOR searches in a direction for a TERMINATOR that induces some type of CHANGE. (Mailhot & Reiss, 2007; Nevins, 2010; Dabbous , 2021)
- We loosen the boolean restriction such that feature functions to have type $\mathcal{D} \rightarrow \{0, -, +\}$.

General Search Structure

$\phi_{f_i}(x) := \text{IF } \iota_i(x) \text{ THEN } \mathbf{s}'_{f_i}(x) \text{ ELSE } f_i(x)$
 $\iota_i(x) := \text{IF } f_i(x) \approx 0 \text{ THEN } \top \text{ ELSE } \perp$
 $\mathbf{s}'_{f_i}(x) := \text{IF } \text{edge}(x) \text{ THEN } \text{def} \text{ ELSE}$
IF $\psi(\delta(x))$ THEN
IF $R_i(\delta(x))$ THEN $f_i(\delta(x))$ ELSE **def**
ELSE $\mathbf{s}'_{f_i}(\delta(x))$

Crimean Tatar Search Structure

$\phi_{\text{rd}}(x) := \text{IF } \iota_{\text{rd}}(x) \text{ THEN } \mathbf{s}_{\text{rd}}(x) \text{ ELSE rd}(x)$
 $\iota_{\text{rd}}(x) := \text{IF } \text{rd}(x) \approx 0 \text{ THEN } \top \text{ ELSE } \perp$
 $\mathbf{s}_{\text{rd}}(x) := \text{IF } \bowtie(p(x)) \text{ THEN } \text{def} \text{ ELSE}$
IF **sy1**($p(x)$) $\wedge$ $\neg$ **rd**($p(x)$) ≈ 0 THEN
IF $\top(p(x))$ THEN **rd**($p(x)$) ELSE **def**
ELSE $\mathbf{s}_{\text{rd}}(p(x))$

TSL Functions with BMRS

- Tier-based strict locality has been argued to be a useful formal characterization of long-distance phenomena. (Heinz , 2011; De Santo & Graf, 2019; Burness , 2021)
- It is possible to define a “tier” predicate with BMRS. We will refer to this as $\tau(x)$.

General Structure for Non-Local Spreading

$\phi_{\mathbf{f}}(x) := \text{IF } \tau(x) \text{ THEN } [\text{IF } \text{edge}_{\tau}(x) \text{ THEN } \mathbf{f}(x) \text{ ELSE } \delta_{\phi_{\mathbf{f}}}^{\tau}(\delta(x))] \text{ ELSE } \mathbf{f}(x)$
 $\text{edge}_{\tau}(x) := \text{IF } \tau(x) \text{ THEN } g(x) \text{ ELSE } \perp$
 $g(x) := \text{IF } \text{edge}(x) \text{ THEN } \top \text{ ELSE IF } \tau(\delta(x)) \text{ THEN } \perp \text{ ELSE } g(\delta(x))$
 $\delta_{\phi_{\mathbf{f}}}^{\tau}(x) := \text{IF } \tau(x) \text{ THEN } \phi_{\mathbf{f}}(x) \text{ ELSE } \delta_{\phi_{\mathbf{f}}}^{\tau}(\delta(x))$

Structure for Crimean Tatar Non-Local [round] Spreading

$\tau(x) := \text{IF } \text{sy1}(x) \text{ THEN } \top \text{ ELSE } \perp$
 $\phi_{\text{rd}}(x) := \text{IF } \tau(x) \text{ THEN } [\text{IF } \text{init}_{\tau}(x) \text{ THEN rd}(x) \text{ ELSE } p_{\phi_{\text{rd}}}^{\tau}(p(x))] \text{ ELSE rd}(x)$
 $\text{init}_{\tau}(x) := \text{IF } \tau(x) \text{ THEN } g(x) \text{ ELSE } \perp$
 $g(x) := \text{IF } \text{init}(x) \text{ THEN } \top \text{ ELSE IF } \tau(p(x)) \text{ THEN } \perp \text{ ELSE } g(p(x))$
 $p_{\phi_{\text{rd}}}^{\tau}(x) := \text{IF } \tau(x) \text{ THEN } \phi_{\text{rd}}(x) \text{ ELSE } p_{\phi_{\text{rd}}}^{\tau}(p(x))$

Input	t	u	z	l	ɯ	ʏ	ɯ
x	1	2	3	4	5	6	7
rd(x)	$\perp$	$\top$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
$\tau(x)$	$\perp$	$\top$	$\perp$	$\perp$	$\top$	$\perp$	$\top$
$\text{init}_{\tau}(x)$	$\perp$	$\top$	$\perp$	$\perp$	$\perp$	$\perp$	$\perp$
$p_{\phi_{\text{rd}}}^{\tau}(x)$	$\perp$	$\top$	$\top$	$\top$	$\top$	$\top$	$\top$
$\phi_{\text{rd}}(x)$	$\perp$	$\top$	$\perp$	$\perp$	$\top$	$\perp$	$\top$

Rendaku (Non-Local Blocking)

- Rendaku is a process in Japanese where the initial obstruent in the second element of a compound is voiced unless there is another voiced obstruent later in the second compound element (Lyman’s Law). (Kawahara, 2015)

Examples

/oo + tako/ $\rightarrow$ [oodako] (big octopus)
/akai + kabu/ $\rightarrow$ [akakabu] (red radish)

- Blocking in the BMRS analysis requires a *covert* SEARCH-like function **VOR**.
Rendaku BMRS Analysis

$\phi_{\text{voice}}(x) := \text{IF } \mathbf{E2I}(x) \text{ THEN } \neg \mathbf{VOR}(x) \text{ ELSE voice}(x)$
 $\mathbf{E2I}(x) := \text{IF } + (p(x)) \text{ THEN } \neg \text{son}(x) \text{ ELSE } \perp$
 $\mathbf{VOR}(x) := \text{IF } \mathbf{VO}(s(x)) \text{ THEN } \top \text{ ELSE IF fin}(x) \text{ THEN } \perp \text{ ELSE } \mathbf{VOR}(s(x))$
 $\mathbf{VO}(x) := \text{IF voice}(x) \text{ THEN } \neg \text{son}(x) \text{ ELSE } \perp$

Tutrugbu (Non-Local Blocking + Spreading)

- Regressive [ATR] harmony is blocked by $[-\text{vowels}]$ if initial vowel is $[+\text{high}]$.

- Spreading can be *overt*(ish) but blocking must be *covert*.

Examples

bu-tí-jě	‘1P-NEG-grow’
e-tí-be-jě	‘3S-NEG-FUT-grow’
bú-tí-bá	‘1P-NEG-come’
a-tí-ba-bá	‘3S-NEG-FUT-come’
i-tí-wu	1S-NEG-climb’
i-ba-wu	1S-FUT-climb’
i-tí-ka-á-ba-ba-wu	1S-FUT-PFV-PROG-VENT-VENT-climb’
e-tí-ke-é-be-be-wu	3S-FUT-PFV-PROG-VENT-VENT-climb’

Tutrugbu [ATR] Harmony BMRS Analysis

(Tier-analysis assumed following Nelson & Baković (2026))

$\text{init/high}(x) := \text{IF } \text{init}(x) \text{ THEN high}(x) \text{ ELSE init/high}(p(x))$
 $\phi_{\text{ATR}}(x) := \text{IF } \text{root}(x) \text{ THEN ATR}(x) \text{ ELSE}$
IF $[-\text{high}](x)$ THEN
IF $\text{init/high}(x)$ THEN $\perp$ ELSE $\phi_{\text{ATR}}(s(x))$
ELSE $\phi_{\text{ATR}}(s(x))$

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