Are Representations in Articulatory and Generative Phonology so different?

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Phonological Background

Representations are central to phonological theory (Anderson, 1985).

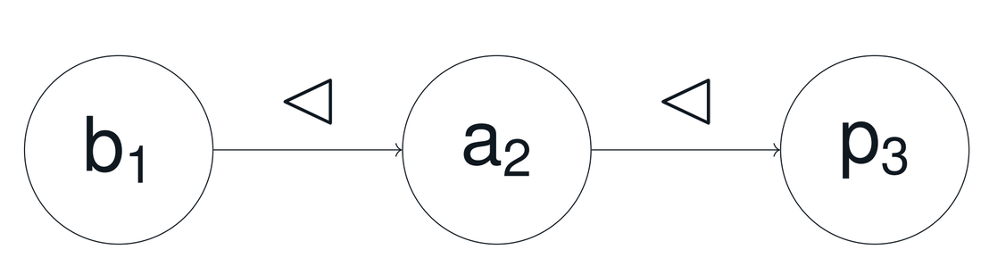
Generative Phonology (Chomsky and Halle, 1968) uses *linear or multilinear symbolic representations* to describe input/output mappings. These structures are interpreted and physically realized by another module of the grammar.

Articulatory Phonology (Browman and Goldstein, 1992) is a theory of phonological representations based around non-linear dynamics which do not have input/output mappings and do not require a separate module for interpretation. Lexical items are represented as *coupling graphs* that dynamically determine a gestural score which describes how articulators form and release constrictions over time (Nam and Saltzman, 2003).

Model Theoretic Phonological Structures

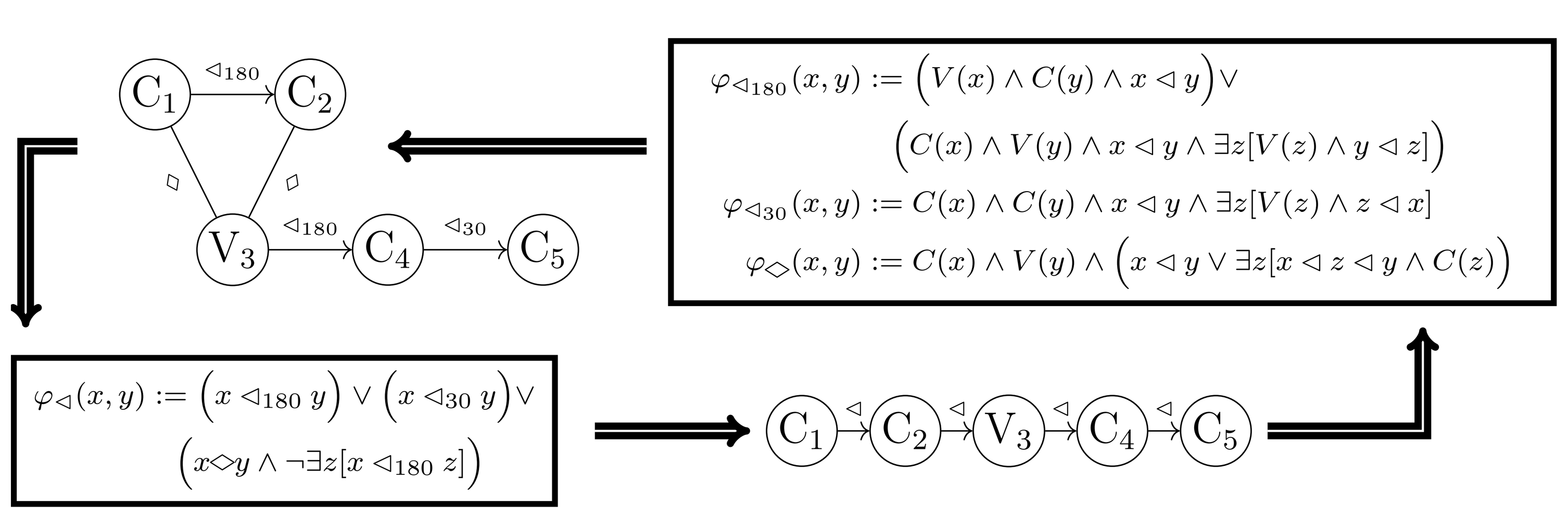
Finite Model Theory can be used to *formally define phonological structures* (Libkin, 2004; Strother-Garcia, 2019; Oakden, 2020; Jardine et al., 2021). **Relational models** include domain elements $\mathcal{D}$ and a set of relations $\mathcal{R}$.

$$\begin{aligned} \langle \mathcal{D} &:= \{1,2,3\} \\ a &:= \{2\} \\ b &:= \{1\} \\ p &:= \{3\} \\ \lhd &:= \{(1,2),(2,3)\}\rangle \end{aligned}$$



MSO Logic Graph Transductions

Translation between representational structures is done using monadic second order logic (Courcelle, 1994). Formulae such as $\varphi_P(x) = Q(x)$ are interpreted as "domain element x has property P in the output structure if it has property Q in the input structure". Additionally, one must specify how many copies of the input domain are needed and which copies are licensed in the output.



$$\varphi_{\lhd_{180}}(x,y) := \Big(V(x) \wedge C(y) \wedge x \lhd y\Big) \vee \Big(C(x) \wedge V(y) \wedge x \lhd y \wedge \exists z[V(z) \wedge y \lhd z]\Big)$$

$$\varphi_{\lhd_{30}}(x,y) := C(x) \wedge C(y) \wedge x \lhd y \wedge \exists z[V(z) \wedge z \lhd x]$$

$$\varphi_{\diamond}(x,y) := C(x) \wedge V(y) \wedge \Big(x \lhd y \vee \exists z[x \lhd z \lhd y \wedge C(z)\Big)$$

$$\varphi_{\lhd}(x,y) := \Big(x \lhd_{180} y\Big) \vee \Big(x \lhd_{30} y\Big) \vee \Big(x \diamond y \wedge \neg\exists z[x \lhd_{180} z]\Big)$$

Bi-Interpretability

Definition (Friedman and Visser, 2014): *We note that an interpretation $K : U \to V$ gives us a construction of an internal model $\widetilde{K}(\mathcal{M})$ of U from a model M of V. We find that U and V are bi-interpretable iff, there are interpretations $K : U \to V$ and $M : V \to U$ and formulas F and G such that, for all models $\mathcal{M}$ of V, the formula F defines an isomorphism between $\mathcal{M}$ and $\widetilde{M}\widetilde{K}(\mathcal{M})$, and, for all models $\mathcal{N}$ of U, the formula G defines an isomorphism between $\mathcal{N}$ and $\widetilde{K}\widetilde{M}(\mathcal{N})$.*

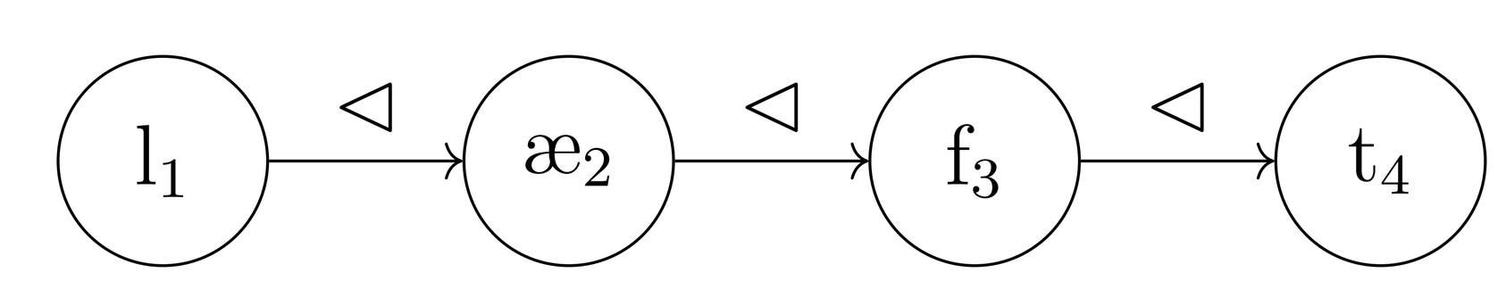
Main Research Question

Are strings and coupling graphs bi-interpretable?

String Model ($\mathcal{M}_s$): [læft]

Relation	Label
$\lhd$	Successor
$\sigma(\forall\sigma \in \Sigma)$	Segment

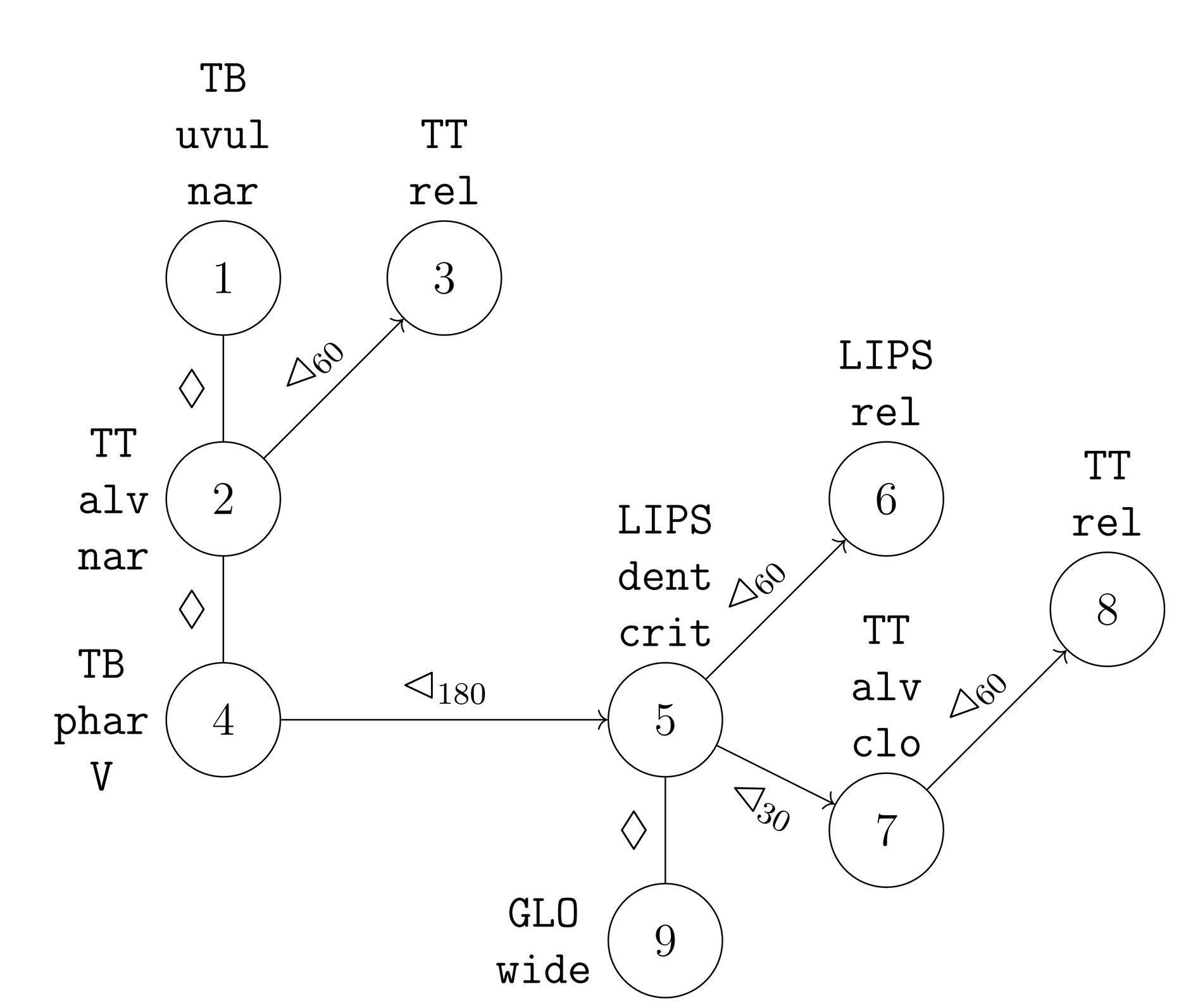
$$\begin{aligned} \langle \mathcal{D} &:= \{1,2,3,4\} \\ \lhd &:= \{(1,2),(2,3),(3,4)\} \\ æ &:= \{2\} \\ f &:= \{3\} \\ l &:= \{1\} \\ t &:= \{4\} \\ x &:= \{\}; x \in \Sigma \setminus \{æ,f,l,t\}\rangle \end{aligned}$$



Coupling Graph Model ($\mathcal{M}_g$): [læft]

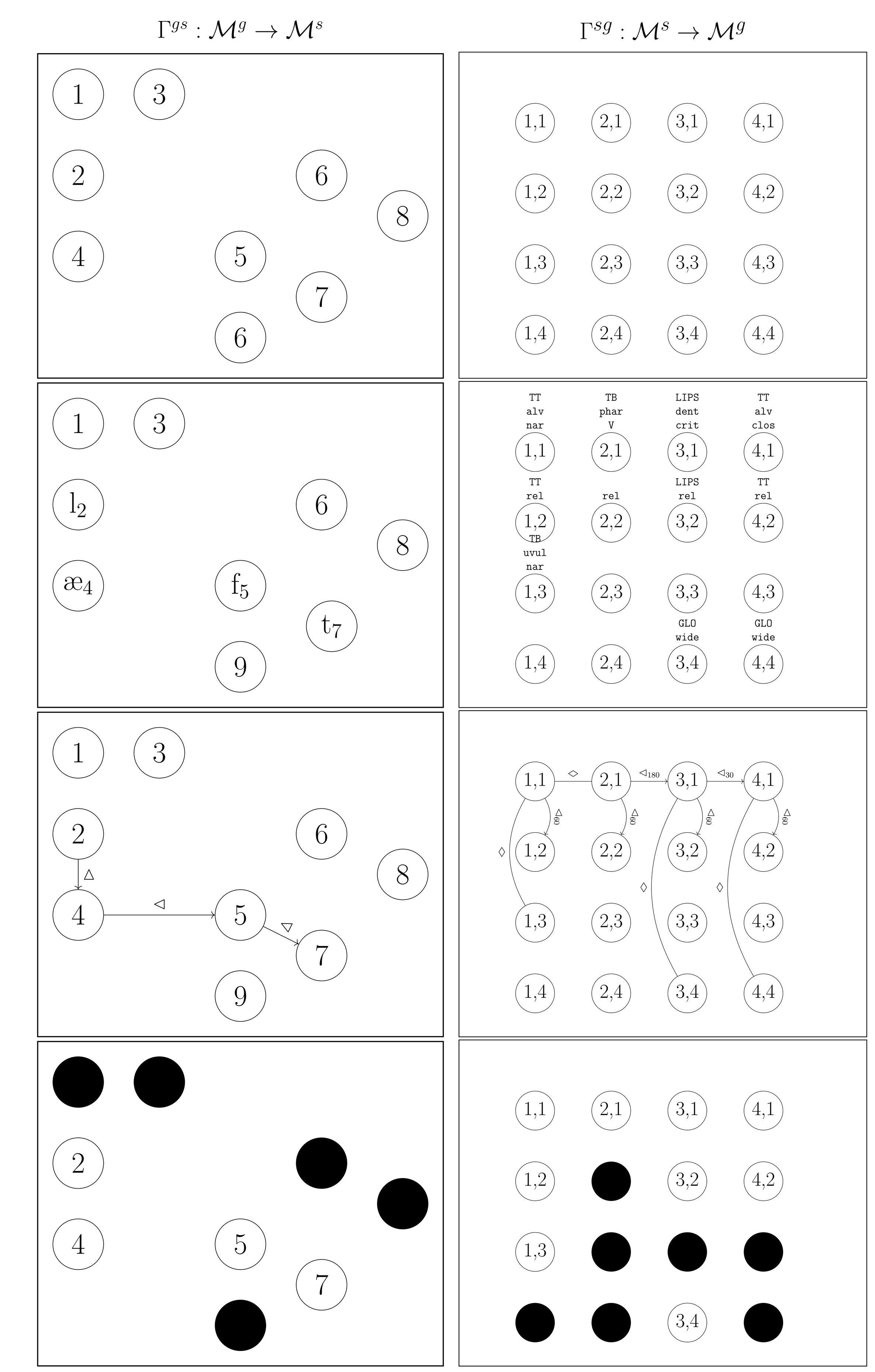
Relation	Label	Relation	Label
$\diamond$	In-phase	$\lhd_{180}$	Anti-phase
$\lhd_{60}$	Abutting	$\lhd_{30}$	Eccentric
LIPS	Labial Articulator	rel	Constriction Degree: release
TT	Tongue Tip Articulator	pro	Constriction Location: protruded
TB	Tongue Body Articulator	dent	Constriction Location: dental
VEL	Velum Articulator	alv	Constriction Location: alveolar
GLO	Glottis Articulator	palv	Constriction Location: postalveolar
clo	Constriction Degree: closed	pal	Constriction Location: palatal
crit	Constriction Degree: critical	vel	Constriction Location: velar
nar	Constriction Degree: narrow	uvul	Constriction Location: uvular
V	Constriction Degree: vowel	phar	Constriction Location: pharyngeal
wide	Constriction Degree: wide		

$$\begin{aligned} \langle \mathcal{D} &:= \{1,2,3,4,5,6,7,8,9\} \\ \diamond &:= \{(1,2),(2,4),(5,9)\} \\ \lhd_{180} &:= \{(4,5)\} \\ \lhd_{60} &:= \{(2,3),(5,6),(7,8)\} \\ \lhd_{30} &:= \{(5,7)\} \\ \texttt{LIPS} &:= \{5,6\} \\ \texttt{TT} &:= \{2,3,7,8\} \\ \texttt{TB} &:= \{1,4\} \\ \texttt{GLO} &:= \{9\} \\ \texttt{dent} &:= \{5\} \\ \texttt{alv} &:= \{2,7\} \\ \texttt{uvul} &:= \{1\} \\ \texttt{phar} &:= \{4\} \\ \texttt{clo} &:= \{7\} \\ \texttt{crit} &:= \{5\} \\ \texttt{nar} &:= \{1,2\} \\ \texttt{wide} &:= \{9\} \\ \texttt{rel} &:= \{3,6,8\} \\ \texttt{V} &:= \{4\}\rangle \end{aligned}$$



Logical Transductions

Box 1: Copies of input domain structure are made
Box 2: Unary relations are determined
Box 3: Binary relations are determined
Box 4: Licit output domain elements are licensed



Conclusion

- Consequently, since $\mathcal{M}_s \equiv \Gamma^{gs}(\Gamma^{sg}(\mathcal{M}_s))$ and $\mathcal{M}_g \equiv \Gamma^{sg}(\Gamma^{gs}(\mathcal{M}_g))$, this indicates that string and coupling graph models are **bi-interpretable**.
- These results also show how logic and model theory provide a shared language to talk about what are often thought to be incompatible theories.